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SPAR DEMONSTRATION PROBLEMS

by

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Corrections attached.

FOREWORD

This report was prepared by Lockheed Missiles and Space Company for the National Aeronautics and Space Administration under Contract NAS8-26352. The contract was funded jointly by NASA's Marshall and Langley Centers, and the U. S. Army Missile Command. The Contracting Officer's representatives were L. A. Kiefling of the MSFC Aero-Astrodynamics Laboratory and J. C. Robinson of the Structures Division at Langley.

The demonstration problems presented in this report were prepared and executed by C. L. Yen and R. A. Moore under the direction of W. D. Whetstone.

INTRODUCTION

A series of examples are presented to indicate some of the principal functions of the SPAR system (Reference 1), and to illustrate SPAR's control card-data card structure. For each of the eleven examples, information in the following categories is given:

- A description of the problem and, in most cases, comparisons with analytical solutions.
- A list of the input cards.
- A printout of the Table of Contents of the direct access library into which all SPAR output was directed.
- A few representative plots.

Many comments are embedded in the input cards. All text to the right of a $ symbol is ignored by SPAR's input decoder. In Problem 1 the input to processors TAB and ELD is extensively annotated. Problem 1 also illustrates the most general form of applied loading. In Problem 10 the INC and MOD commands are extensively used in ELD input. Problem 11 illustrates the use of a spectral shift in solving an eigenproblem.

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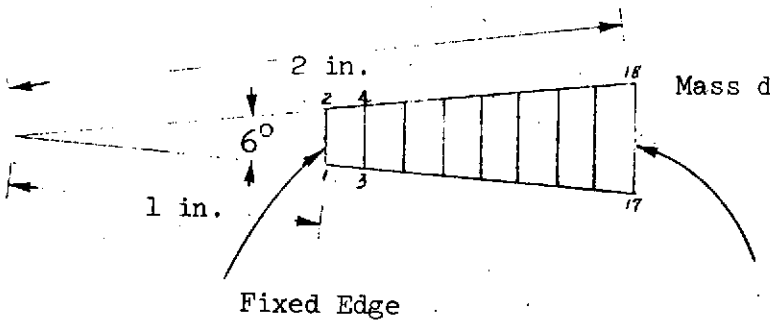
REFERENCES

1. RING SECTOR

Solutions were obtained for a segment of a ring in order to explain in detail various data for processors TAB and ELD, and to demonstrate the use of combined loadings in one load case definition in the Q processor. A 6-degree sector of a ring was divided into an 8 x 1 quadrilateral finite element mesh. The geometrical and material properties of the ring are shown on the figure below:



$E = 10.0$ lb $/in^2$
$\nu = 0.3$
Mass density $= 1.0$ lb-sec$^2/$in^4
$\alpha = 1.0$ in/degree
$t = 1.0$ in

Linear and quadratic thermal loads were applied to the ring, as well as a combination loading of nodal forces, thermal loads, specified nodal motion, surface pressure, and an inertia load.

```
@XQT TAB                                . GENERATE BASIC TABLES DEFINING STRUCTURE
$                                         STRUCTURE HAS 18 NODES
$                                         DEGREES OF FREEDOM 1,2,3,4,5 ARE ACTIVE
START 18   6$
$                                         TITLE DESIGNATION TO BE ASSOCIATED WITH
$                                         DIRECT ACCESS LIBRARY (DAL)
TITLE!   RING WITH COMBINATION LOADINGS
$                                         EXPLANATORY TEXT MATERIAL TO BE DISPLAYED
TEXT$
!A RING UNDER LINEAR/QUADRATIC TEMPERATURE LOADING AND COMBINED LOADINGS
MATERIAL CONSTANTS$
$                                         ENTRY 1 OF MATERIAL PROPERTIES IN WHICH
$                                         MODULUS OF ELASTICITY = 10.0
$                                         POISSONS RATIO = 0.3
$                                         MASS DENSITY = 1.0
$                                         THERMAL EXPANSION COEFFICIENT = 1.0
      1 10. .3 1. 1.$
JOINT LOCATIONS$
$                                         NODAL LOCATIONS DEFINED IN CYLINDRICAL
$                                         COORDINATES
      FORMAT=2$
$                                         NODE 1 HAS COORDINATES R=1.0, THETA=0.0,
$                                         AND Z=0.0
$                                         NODE 17 HAS COORDINATES R=2.0, THETA=0.0,
$                                         AND Z=0.0
$                                         LINEAR INTERPOLATION OF COORDINATES
$                                         BETWEEN NODES 1 AND 17 FOR NODES
$                                         3,5,7,9,11,13 AND 15
      1 1. 0. 0. 2. 0. 0. 9 2 2$
$                                         NODE 2 HAS COORDINATES R=1.0, THETA=6.0
$                                         AND Z=0.0
$                                         NODE 18 HAS COORDINATES R=2.0, THETA=6.0,
$                                         AND Z=0.0
$                                         LINEAR INTERPOLATION OF COORDINATES
$                                         BETWEEN NODES 2 AND 18 FOR NODES
$                                         4,6,8,10,12,14 AND 16
      1 1. 6. 0. 2. 6. 0.$
JOINT REFERENCE FRAMES$
$                                         ALL JOINT REFERENCE FRAMES WILL BE
$                                         TANGENT TO GLOBAL CYLINDRICAL COORDINATE
$                                         LINES
      NREF=-1$
      1 18 $
SHELL SECTION PROPERTIES$
$                                         ENTRY 1 IS ISOTROPIC SECTION WITH T=1.0
      1 1.$
CONSTRAINT CASE 1$
$                                         CONSTRAIN ALL DEGREES OF FREEDOM AT ONE
$                                         END.  CONSTRAINTS AND SPECIFIED MOTIONS
$                                         AT THE OTHER END.
      ZERO 1 2 3 4 5$ 1 2$
      ZERO 1 2 4 5  $ 17 18$
      NONZERO 3     $ 17 18$
@XQT ELD                                . READ ELEMENT DEFINITIONS
E43 $
$                                         DEFINE A QUADRILATERAL MEMBRANE/BENDING
$                                         NETWORK, ELEMENT NODE NUMBERS: 1 3 4 2,
$                                         3 5 6 4, .... (A TOTAL OF 8 ELEMENTS,
$                                         THE LAST OF WHICH IS 15 17 18 16)
      1 3 4 2   1 8 1$
@XQT TOPO                               . ANALYZE ELEMENT INTERCONNECTIVITY
```


REPRODUCIBILITY OF THE
ORIGINAL PAGE IS POOR

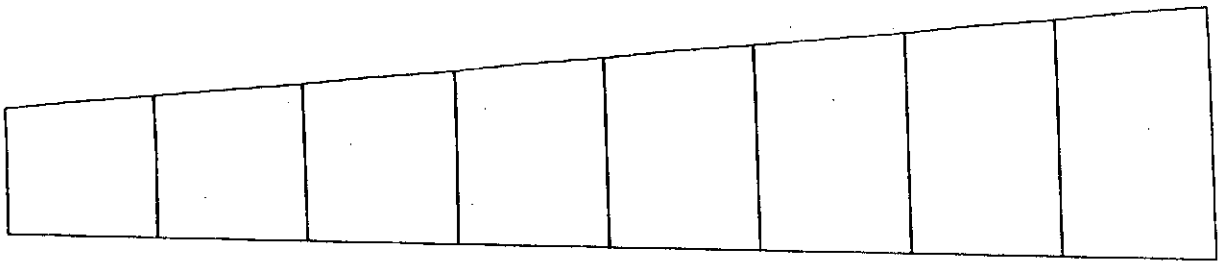
```
@XQT E                                  • FORM ELEMENT DATA PACKETS
@XQT EKS                                • INSERT K, S INTO ELEMENT DATA PACKETS
@XQT K                                  • FORM SYSTEM K
@XQT INV                                • FACTOR MATRIX IN K SPAR FORMAT
@XQT M                                  • FORM SYSTEM M
@XQT Q                                  • DEFINE APPLIED LOADS
     RESET MLIB=1$
     CASE 1(LINEAR RADIAL TEMPERATURE LOAD
       NODAL TEMPERATURES
$                                       GENERATE A THERMAL LOADING WITH
$                                       THE THERMAL EXPANSION COEFFICIENT = 1.
$                                       FOR THE MATERIAL DEFINED
$                                       NODES 3,5,7,9,11,13,15   LOAD 0.5
$                                       NODES 4,6,6,10,12,14,16  LOAD 0.0625
          3 .5 7 2 .625=1 2 1$
     CASE 2(QUADRATIC RADIAL TEMPERATURE LOAD
       NODAL TEMPERATURES
$                                       GENERATE A SERIES OF THERMAL LOADS
$                                       NODES  3  4  LOAD 0.3164
$                                       NODES  5  6  LOAD 0.3906
$                                         .
$                                         .
$                                         .
$                                       NODES 15 16  LOAD 0.6787
          3 .3164  2 1$
          5 .3906  2 1$
          7 .4727  2 1$
          9 .5625  2 1$
         11 .6602  2 1$
         13 .7656  2 1$
         15 .8787  2 1$
     CASE 3(COMBINED LOADING
       NODAL FORCES, MOMENTS
$                                       LOAD ON NODE  9 IN NORMAL DIRECTION,
$                                       MAGNITUDE=-1.0
$                                       LOAD ON NODE 10 IN NORMAL DIRECTION,
$                                       MAGNITUDE= 1.0
          9 3 -1.$
         10 3  1.$
       NODAL TEMPERATURES
$                                       UNIFORM TEMPERATURE LOAD ALONG ONE EDGE
$                                       NODES 3,5,7,9,11,13,15  MAGNITUDE=1.
          3 1. 7 2$
       NODAL MOTIONS
$                                       SPECIFIED MOTION AT ONE END
$                                       NODES 17 18,  MAGNITUDE=0.1
$                                       DIRECTION 3 (SPECIFIED AS NONZERO
                                        MOTION ON THE CONSTRAINT CASE)
         17 3 .1 2 1$
       NODAL PRESSURES
$                                       UNIFORM PRESSURE ACROSS THE SURFACE
$                                       NODES 3,15,  MAGNITUDE=1.0
          3 1, 2 1 0, 7 2$
$                                       GRAVITY LOAD NORMAL TO THE SURFACE
$                                       MAGNITUDE=-32.2 OR -1 G
       INERTIA FORCE 3 -32.2$
@XQT DSOL                               • COMPUTE STATIC SOLUTIONS
$                                       PRINT NODAL DISPLACEMENTS AND REACTION
$                                       FORCES FOR ALL SOLUTION CASES
     RESET UP=1,FP=1$
@XQT GSF                                • GENERATE STRESS DATA
@XQT PSF                                • PRINT STRESS DATA
@XQT DCU                                • EXECUTE DATA COMPLEX UTILITY PROGRAM
     TOC 1$
```

TABLE OF CONTENTS, DAL 1

RING WITH COMBINATION LOADINGS

SEQ	RR	DATE	TIME	F R	WORDS	ROWS /BLK	BLK SIZE	T Y	DATA SET NAME N1	N2	N3	N4
1	11	060574	114309	0	18	1	18	0	JDF1	BTAB	1	8
2	12	060574	114309	0	18	18	18	0	JREF	BTAB	2	6
3	13	060574	114309	0	12	1	12	1	ALTR	BTAB	2	4
4	14	060574	114309	0	19	1	19	4	NDAL		0	0
5	15	060574	114310	0	20	1	20	3	TEXT	BTAB	2	1
6	16	060574	114310	0	10	1	10	1	MATC	BTAB	2	2
7	17	060574	114311	0	54	18	54	1	JLOC	BTAB	2	5
8	19	060574	114311	0	18	18	18	0	JREF	BTAB	2	6
9	20	060574	114312	0	19	1	19	1	SA	BTAB	2	13
10	21	060574	114312	0	18	18	18	0	CON		1	0
11	22	060574	114312	0	162	18	162	1	QJJT	BTAB	2	19
12	28	060574	114315	0	144	9	896	0	DEF	E43	11	4
13	60	060574	114315	0	2	1	2	0	GD	E43	11	4
14	61	060574	114315	0	15	1	15	0	GTIT	E43	11	4
15	62	060574	114315	0	12	12	12	0	NELZ	BTAB	1	11
16	63	060574	114315	0	5	1	5	0	HE		0	0
17	64	060574	114315	0	6	1	6	0	NS		0	0
18	65	060574	114315	0	1	1	1	3	ELTS	NAME	0	0
19	66	060574	114315	0	1	1	1	0	ELTS	LTYP	0	0
20	67	060574	114315	0	1	1	1	0	ELTS	NNOD	0	0
21	77	060574	114315	0	1	1	1	0	ELTS	ISCT	0	0
22	78	060574	114315	0	1	1	1	0	ELTS	NELS	0	0
23	79	060574	114315	0	1	1	1	0	ELTS	LES	0	0
24	80	060574	114318	0	1344	18	1344	0	KMAP		59	10
25	128	060574	114319	0	1344	18	1344	0	AMAP	0000	59	10
26	176	060574	114324	0	2772	9	308	4	E43		11	4
27	275	060574	114321	0	20	20	20	0	DIR	E43	11	4
28	276	060574	114326	0	2240	18	2240	1	K	SPAR	25	59
29	356	060574	114330	0	3136	18	3136	1	INV	K	1	59
30	468	060574	114334	0	2240	18	2240	1	M	SPAR	25	59
31	548	060574	114339	0	108	18	108	-1	RBMX	VEC	1	0
32	552	060574	114339	0	108	18	108	-1	RBMX	VEC	2	0
33	556	060574	114339	0	108	18	108	-1	RBMX	VEC	3	0
34	560	060574	114339	0	18	18	18	-1	NTMP		0	1
35	561	060574	114340	0	108	18	108	-1	NFM		0	1
36	565	060574	114340	0	15	1	15	4	CASE	0000	0	1
37	566	060574	114340	0	18	18	18	-1	NTMP		0	2
38	567	060574	114341	0	108	18	108	-1	NFM		0	2
39	571	060574	114341	0	15	1	15	4	CASE	0000	0	2
40	572	060574	114342	0	18	18	18	-1	NTMP		0	3
41	582	060574	114342	0	108	18	108	-1	NMOT		0	3
42	586	060574	114343	0	108	18	108	-1	NFM		0	3
43	590	060574	114343	0	15	1	15	4	CASE	0000	0	3
44	591	060574	114343	0	0	0	0	0	LDIR		1	3
45	591	060574	114350	0	108	18	108	1	SSOL	U	1	1
46	595	060574	114351	0	108	18	108	1	SSOL	U	1	2
47	599	060574	114352	0	108	18	108	1	SSOL	U	1	3
48	603	060574	114358	0	441	9	5600	-1	STRS	E43	0	1
49	803	060574	114400	0	441	9	5600	-1	STRS	E43	0	2
50	1003	060574	114400	0	441	9	5600	-1	STRS	E43	0	3

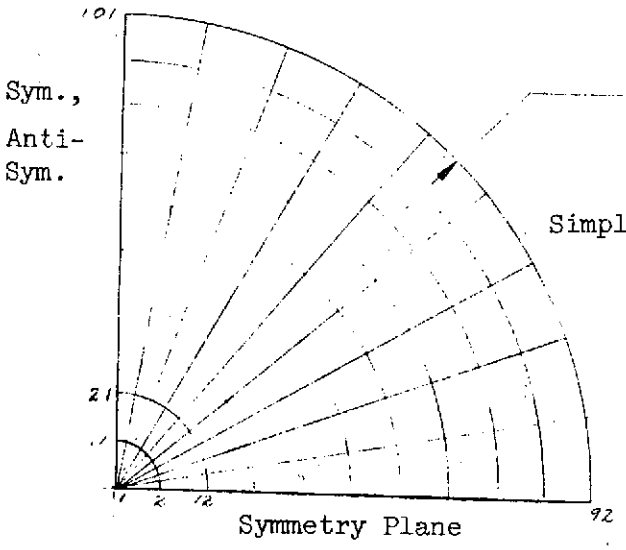
UNDEFORMED STRUCTURE



RING SECTOR

2. VIBRATION OF A CIRCULAR MEMBRANE

Natural frequencies were obtained for lateral vibration of a simply supported circular membrane in uniform tension. A quarter of the membrane was divided into 9 x 10 finite elements to obtain the numerical solution. The geometrical and material properties of the membrane are shown on the figure below:



$E = .3 \times 10^8$ lb/in^2
$\nu = .3$ lb/in^2
Mass density $= .1$ lb-sec^2/in^4
$t = .1$ in

Numerical results are listed below:

	SPAR	Analytical
ω_{01}^2	$.58622 \times 10$	$.57792 \times 10$
ω_{11}^2	$.15003 \times 10^2$	$.14684 \times 10^2$
ω_{21}^2	$.26999 \times 10^2$	$.26337 \times 10^2$
ω_{02}^2	$.31389 \times 10^2$	$.30471 \times 10^2$
ω_{31}^2	$.41982 \times 10^2$	$.40692 \times 10^2$
ω_{12}^2	$.51580 \times 10^2$	$.49224 \times 10^2$

```
   @XQT TAB                                  . GENERATE BASIC TABLES DEFINING STRUCTURE
   START 101,4 5 6$
   TITLE! VIBRATION OF A CIRCULAR MEMBRANE
   MATERIAL CONSTANTS$
        1 3.+7 .3 .1 1.-5 1.+4$
   JOINT REFERENCE FRAME ASSIGNMENTS$
        NREF=-1; 1 101 1$
   JOINT LOCATIONS$
        FORMAT=2; 1 0. 0. 0.; 2 1. 0. 0. 1. 90. 0. 10 1 10; 10 10. 0. 0. 10. 90. 0.$
   SHELL SECTION PROPERTIES$
        1 .1$
   CONSTRAINT 1$
   $                                         CONSTRAINED AT EDGES AND SYMMETRY AT
   $                                         THETA=0 AND 90 DEGREE PLANES
        SYMMETRY PLANE=2$
        SYMMETRY PLANE=1$
        ZERO 1 2 3; 92 101 1$
   CONSTRAINT 2$
   $                                         CONSTRAINED AT EDGES. SYMMETRY AT THETA=0
   $                                         DEGREE PLANE. ANTI-SYMMETRY AT THETA=90
   $                                         DEGREE PLANE
        ANTISYMMETRY PLANE=1$
        SYMMETRY PLANE =2$
        ZERO 1; 1$
        ZERO 1 2 3; 92 101 1$
   CONSTRAINT 3$
   $                                         SAME AS CASE 1 EXCEPT RADIAL DIRECTION IS
   $                                         FREE AT EDGES.
        SYMMETRY PLANE=1$
        SYMMETRY PLANE=2$
        ZERO 2 3; 1 101 1$
   @XQT ELD                                  . READ ELEMENT DEFINITIONS
   E41 $
        2 12 13 3     1 9 9$
   E31 $
        1 2 3     2 9 1$
   @XQT TOPO                                 . ANALYZE ELEMENT INTERCONNECTIVITY
   $
   @XQT E                                    . FORM ELEMENT DATA PACKETS
   $
   @XQT EKS                                  . INSERT K, S INTO ELEMENT DATA PACKETS
   $
   @XQT K                                    . FORM SYSTEM K
   @XQT INV                                  . FACTOR MATRIX IN K SPAR FORMAT
        RESET CON=3$
   @XQT M                                    . FORM SYSTEM M
   $
   @XQT Q                                    . DEFINE APPLIED LOADS
        CASE 1! RADIAL TENSION AT EDGES
        NODAL FORCES, MOMENTS$
        92 1 .872665 2 9; 93 1 1.74533 8 1$
   @XQT DSOL                                 . COMPUTE STATIC SOLUTIONS
        RESET CON=3$
   @XQT KG                                   . FORM SYSTEM KG, UNIFORM TENSION
   @XQT LCM                                  . FORM LINEAR COMBINATION OF MATRICES
   $                                         . FORM STIFFNESS MATRIX FOR RADIALLY
   $                                         STRESSED CIRCULAR MEMBRANE
   @XQT INV
        RESET K=K+KG$
   @XQT EIG                                  . SOLVE SYSTEM EIGENPROBLEM
   $                                         FIND VIBRATIONAL MODES OF THE STRESSED
```


REPRODUCIBILITY OF THE
ORIGINAL PAGE IS POOR

```
$                                          CIRCULAR MEMBRANE UNDER CONSTRAINT CASE 1
    RESET K=K+KG,INIT=8$
PRINT 0 0 0 0 1$
*XQT INV
    RESET K=K+KG,CON=2$
*XQT EIG
$                                          FIND VIBRATIONAL MODES OF THE STRESSED
$                                          CIRCULAR MEMBRANE UNDER CONSTRAINT CASE 2
    RESET K=K+KG,INIT=8,CON=2$
    PRINT 0 0 0 0 -1$
*XQT DCU                                   . EXECUTE DATA COMPLEX UTILITY PROGRAM
    TOC 1$
```

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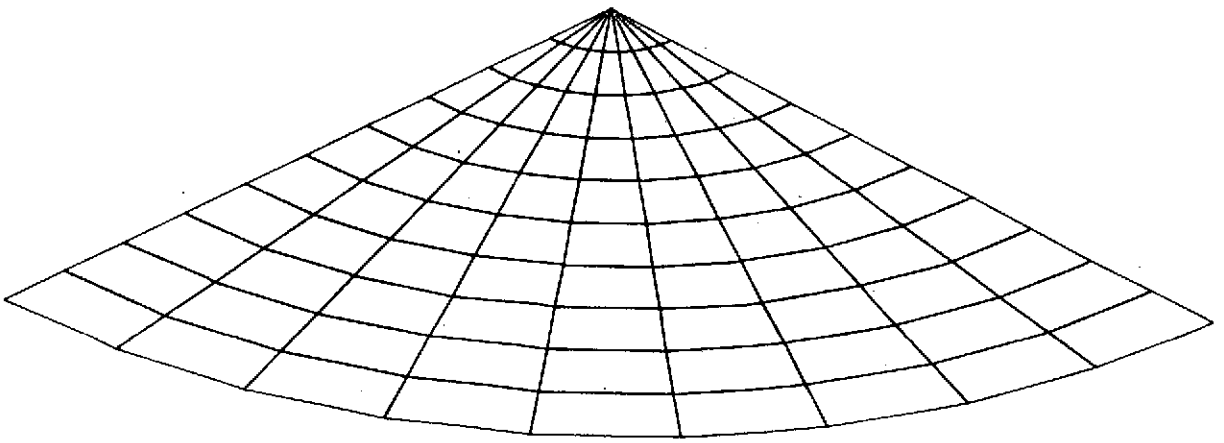
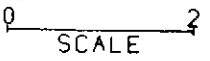
VIBRATION OF A CIRCULAR MEMBRANE

SEQ	RR	DATE	TIME	ER	WORDS	ROWS /BLK	BLK SIZE	TY	DATA SET NAME N1	N2	N3	N4
1	11	051474	042706	0	18	1	18	0	JDF1	BTAB	1	8
2	-12	051474	042706	0	101	101	101	0	JREF	BTAB	2	6
3	16	051474	042706	0	12	1	12	1	ALTR	BTAB	2	4
4	17	051474	042706	0	19	1	19	4	NDAL		0	0
5	18	051474	042706	0	10	1	10	1	MATC	BTAB	2	2
6	19	051474	042706	0	101	101	101	0	JREF	BTAB	2	6
7	23	051474	042707	0	303	101	303	1	JLOC	BTAB	2	5
8	34	051474	042707	0	19	1	19	1	SA	BTAB	2	13
9	35	051474	042707	0	101	101	101	0	CON.		1	0
10	39	051474	042708	0	101	101	101	0	CON		2	0
11	43	051474	042709	0	101	101	101	0	CON		3	0
12	47	051474	042709	0	909	101	909	1	GJJT	BTAB	2	19
13	80	051474	042715	0	1296	81	896	0	DEF	E41	9	4
14	144	051474	042715	0	2	1	2	0	GD	E41	9	4
15	145	051474	042715	0	15	1	15	0	GTIT	E41	9	4
16	146	051474	042715	0	144	9	896	0	DEF	E31	6	3
17	178	051474	042715	0	2	1	2	0	GD	E31	6	3
18	179	051474	042715	0	15	1	15	0	GTIT	E31	6	3
19	180	051474	042715	0	12	12	12	0	NELZ	BTAB	1	11
20	181	051474	042715	0	10	2	10	0	KE		0	0
21	191	051474	042715	0	12	2	12	0	NS		0	0
22	192	051474	042716	0	2	2	2	3	ELTS	NAME	0	0
23	193	051474	042716	0	2	2	2	0	ELTS	LTYP	0	0
24	194	051474	042717	0	2	2	2	0	ELTS	NNOD	0	0
25	195	051474	042717	0	2	2	2	0	ELTS	ISCT	0	0
26	196	051474	042718	0	2	2	2	0	ELTS	NELS	0	0
27	197	051474	042718	0	2	2	2	0	ELTS	LE3	0	0
28	198	051474	042721	0	2688	101	1344	0	KMAP		453	10
29	294	051474	042726	0	9408	101	1344	0	AMAP	@@@1	1137	78
30	630	051474	042734	0	1260	9	140	4	E31		6	3
31	675	051474	042730	0	20	20	20	0	DIR	E31	6	3
32	676	051474	042736	0	13608	81	168	4	E41		9	4
33	1162	051474	042732	0	20	20	20	0	DIR	E41	9	4
34	1163	051474	042743	0	6720	101	2240	1	M	SPAR	9	453
35	1403	051474	042745	0	606	101	606	-1	NFM		0	1
36	1425	051474	042745	0	15	1	15	4	CASE	@@@@	0	1
37	1426	051474	042745	0	0	0	0	0	LDIR		1	1
38	1426	051474	042751	0	6720	101	2240	1	K	SPAR	9	453
39	1666	051474	042755	0	9408	101	3136	1	INV	K	3	1137
40	2002	051474	042758	0	606	101	606	1	SSOL	U	0	1
41	2033	051474	042806	0	6720	101	2240	1	KG	SPAR	9	453
42	2273	051474	042807	0	6720	101	2240	1	K*KG	SPAR	9	453
43	2513	051474	042815	0	12544	101	3136	1	INV	K*KG	1	1137
44	2961	051474	042816	0	8	8	8	-1	VIBR	EVAL	1	0
45	2962	051474	042817	0	4848	101	606	-1	VIBR	U	1	0
46	3138	051474	042919	0	12544	101	3136	1	INV	K*KG	2	1137
47	3586	051474	042921	0	8	8	8	-1	VIBR	EVAL	2	0
48	3587	051474	042922	0	4848	101	606	-1	VIBR	U	2	0

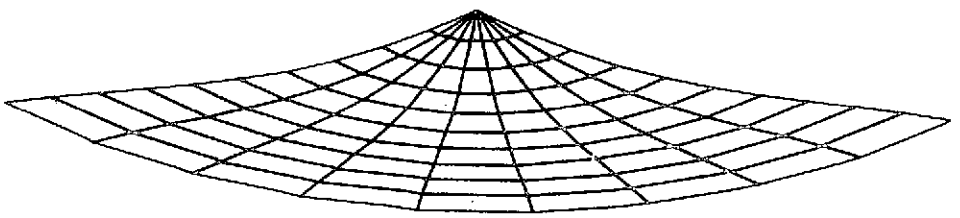
UNDEFORMED STRUCTURE



CIRCULAR MEMBRANE

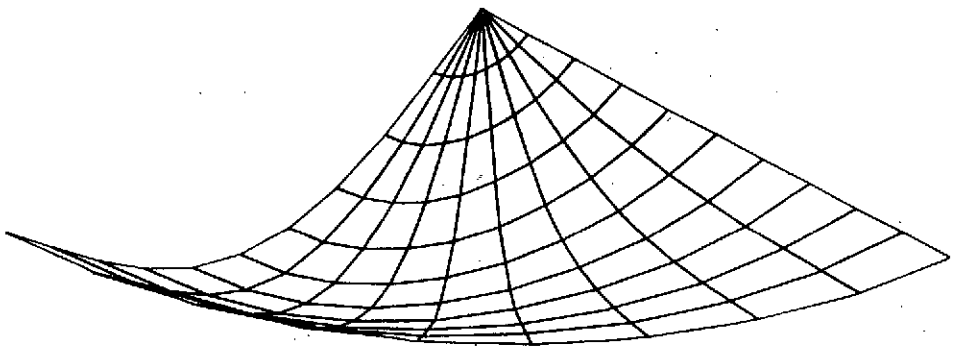


SEQ 1 VIBRATIONAL MODE, FREQ (HZ) = .385344X10^+00



CIRCULAR MEMBRANE

SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = .616456X10^{+00}

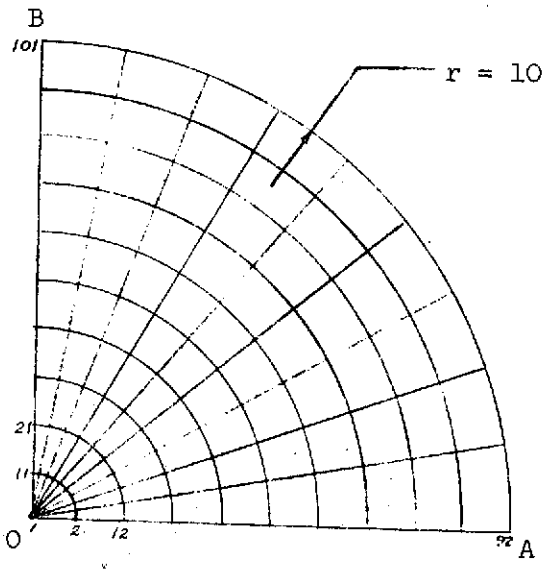


CIRCULAR MEMBRANE

0 3
SCALE

3. CIRCULAR PLATE

The quarter-circle finite element mesh shown on Figure 3-1 was used to solve the problems described in Sections 3.1, 3.2, and 3.3.



$E = .3 \times 10^8$ lb /in^2

$\nu = .3$

Mass density = .1 lb-sec^2/in^4

$t = .1$ in

Fig. 3-1 Circular Plate

3.1 Vibrational Characteristics. With edge AB fixed, and edge OA a symmetry plane, solutions were obtained for (1) OB a symmetry plane, and (2) OB an anti-symmetry plane. SPAR results are compared with Reference 2, pp 449, 450.

	OB = Symmetry Plane		OB = Anti-Symmetry Plane	
	ω_{10}^2	ω_{12}^2	ω_{20}^2	ω_{11}^2
SPAR	$.28991 \times 10^4$	$.33770 \times 10^5$	$.43923 \times 10^5$	$.12547 \times 10^5$
Ref. 2	$.28638 \times 10^4$	$.33470\ \ 10^5$	$.43474 \times 10^5$	$.12371 \times 10^5$

3.2 Buckling due to Compressive Edge Load. Both OA and OB are symmetry planes. Unit uniform compression is developed by applying $N_r = 1$ along free

edge AB. With edge AB then clamped, N_r (critical) is computed. The result indicated below is compared with Reference 3, pp 389-392.

	N_r (Critical)
SPAR	$.40298 \times 10^3$
Ref. 3	$.40326 \times 10^3$

3.3 Lateral Deflection, including Prestress Effects. OA and OB are symmetry planes. Uniform tension is developed by applying $N_R = 100$ along free edge AB. AB is then clamped, and unit uniform lateral pressure is applied. The result indicated below is compared with Reference 4, pp 391-393.

	Lateral Deflection at Center
SPAR	$.75739 \times 10^{-1}$
Ref. 4	$.75960 \times 10^{-1}$

```
@XQT TAB                                    . GENERATE BASIC TABLES DEFINING STRUCTURE
START 101,6$
TITLE!  CIRCULAR PLATE PROBLEMS
TEXT$
!      THE FOLLOWING CIRCULAR PLATE PROBLEMS ARE SOLVED IN THIS RUN!
!       1  VIBRATION OF A CIRCULAR PLATE
!       2  BUCKLING OF A CIRCULAR PLATE
!       3  LATERAL DEFLECTION OF A PRESTRESSED CIRCULAR PLATE
!
MATEIRAL CONSTANTS$
    1 .3+8  .3  .1  .1-4$
JOINT LOCATIONS$
    FORMAT=2$
      1   .0  .0  .0
      2  1.0  .0  .0   1.0 90.0  .0  10 1 10
     10 10.0  .0  .0  10.0 90.0  .0
JOINT REFERENCE FRAME ASSIGNMENTS$
    NREF=-1;  1,101$
SHELL SECTION PROPERTIES
      1 0.1$
CONSTRAINT CASE 1$
$                                           CLAMPED AT EDGES, SYMMETRY AT BOTH THETA=
$                                           0 AND 90 DEGREE PLANES
    SYMMETRY PLANE=2$
    SYMMETRY PLANE=1$
    ZERO 2,3,4,5; 92,101$
CONSTRAINT CASE 2$
$                                           CLAMPED AT EDGES, SYMMETRY AT THETA=0
$                                           PLANE AND ANTI-SYMMETRY AT THETA=90
$                                           DEGREE PLANE.
    ANTISYMMETRY PLANE=1$
    SYMMETRY PLANE=2$
    ZERO 1,2,3,4,5; 92,101$
    ZERO 1; 1$
@XQT ELD                                    . READ ELEMENT DEFINITIONS
E33 $
    1 2 3  2 9$
E43 $
    2 12 13 3  1 9 9$
@XQT TOPO                                   . ANALYZE ELEMENT INTERCONNECTIVITY
@XQT E                                      . FORM ELEMENT DATA PACKETS
$
@XQT EKS                                    . INSERT K, S INTO ELEMENT DATA PACKETS
$
@XQT M                                      . FORM SYSTEM M
$
@XQT K                                      . FORM SYSTEM K
$
@XQT INV                                    . FACTOR MATRIX IN K SPAR FORMAT
$
@XQT EIG                                    . SOLVE SYSTEM EIGENPROBLEM
$                                           FIND VIBRATIONAL MODES OF THE CIRCULAR
$                                           PLATE UNDER CONSTRAINT CONDITION 1
    RESET INIT=8$
@XQT INV
    RESET CON=2
@XQT EIG
$                                           FIND VIBRATIONAL MODES OF THE CIRCULAR
$                                           PLATE UNDER CONSTRAINT CONDITION 2
    RESET INIT=8,CON=2$
@XQT Q                                      . DEFINE APPLIED LOADS
```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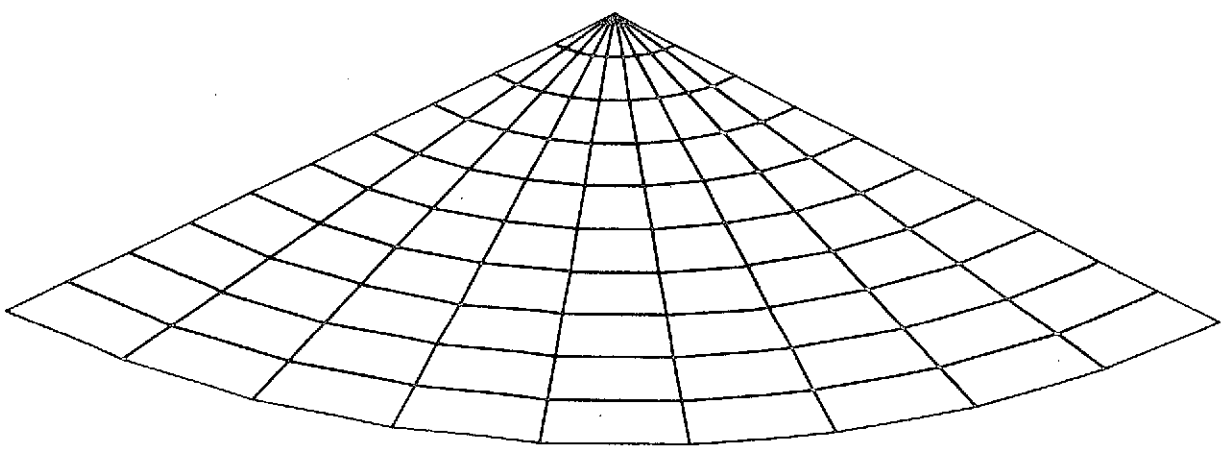
```
     CASE 1: UNIFORM RADIAL COMPRESSIVE FORCE AT EDGES
       NODAL FORCES, MOMENTS
         92 1 -0.8726463   2 9$
         93 1 -1.7453293   8 1$
     CASE 2: UNIFORM LATERAL PRESSURE LOAD
       NODAL PRESURE
         1  1.0  101 1$
@XQT DSOL                                  . COMPUTE STATIC SOLUTIONS
     RESET L1=1,L2=1$
@XQT KG                                    . FORM SYSTEM KG
@XQT EIG
$                                            FIND CRITICAL RADIAL COMPRESSIVE LOAD.
     RESET PROB=STAB,INIT=4,UPPER=1,NDYN=10$
@XQT LCM                                   . FORM LINEAR COMBINATION OF MATRICES
$                                            FORM STIFFNESS MATRIX FOR PRE-COMPRESSED
$                                            CIRCULAR PLATE, COMPRESSIVE FORCE=100.
     RESET R=KMKG,Q1=K,Q2=KG,C2=100.0$
@XQT INV
     RESET K=KMKG$
@XQT DSOL
$                                            FIND LATERAL DEFLECTION OF PRE-COMPRESSED
$                                            CIRCULAR PLATE UNDER UNIFORM PRESSURE
     RESET K=KMKG,L1=2,L2=2,UP=1$
@XQT DCU                                   . EXECUTE DATA COMPLEX UTILITY PROGRAM
     TOC 1$
```

CIRCULAR PLATE PROBLEMS

SEQ	RR	DATE	TIME	ER	WORDS	ROWS /BLK	BLK SIZE	TY	DATA SET NAME N1	N2	N3	N4
1	11	051374	213904	0	18	1	18	0	JDF1	BTAB	1	8
2	12	051374	213904	0	101	101	101	0	JREF	BTAB	2	6
3	16	051374	213904	0	12	1	12	1	ALTR	BTAB	2	4
4	17	051374	213904	0	19	1	19	4	NDAL		0	0
5	18	051374	213904	0	80	4	80	3	TEXT	BTAB	2	1
6	21	051374	213905	0	10	1	10	1	MATC	BTAB	2	2
7	22	051374	213905	0	303	101	303	1	JLOC	BTAB	2	5
8	33	051374	213905	0	101	101	101	0	JREF	BTAB	2	6
9	37	051374	213906	0	19	1	19	1	SA	BTAB	2	13
10	38	051374	213906	0	101	101	101	0	CON		1	0
11	42	051374	213907	0	101	101	101	0	CON		2	0
12	46	051374	213907	0	909	101	909	1	QJJT	BTAB	2	19
13	79	051374	213911	0	144	9	896	0	DEF	E33	8	3
14	111	051374	213911	0	2	1	2	0	GD	E33	8	3
15	112	051374	213911	0	15	1	15	0	GTIT	E33	8	3
16	113	051374	213912	0	1296	81	896	0	DEF	E43	11	4
17	177	051374	213912	0	2	1	2	0	GD	E43	11	4
18	178	051374	213912	0	15	1	15	0	GTIT	E43	11	4
19	179	051374	213912	0	12	12	12	0	NELZ	BTAB	1	11
20	180	051374	213912	0	10	2	10	0	KE		0	0
21	190	051374	213913	0	12	2	12	0	NS		0	0
22	191	051374	213913	0	2	2	2	3	ELTS	NAME	0	0
23	192	051374	213913	0	2	2	2	0	ELTS	LTYP	0	0
24	193	051374	213913	0	2	2	2	0	ELTS	NNOD	0	0
25	194	051374	213914	0	2	2	2	0	ELTS	ISCT	0	0
26	195	051374	213914	0	2	2	2	0	ELTS	NELS	0	0
27	196	051374	213915	0	2	2	2	0	ELTS	LE3	0	0
28	197	051374	213918	0	2688	101	1344	0	KMAP		453	30
29	293	051374	213921	0	9408	101	1344	0	AMAP	@@@@	1137	78
30	629	051374	213930	0	2016	9	224	4	E33		8	3
31	701	051374	213924	0	20	20	20	0	DIR	E33	8	3
32	702	051374	213937	0	24948	81	308	4	E43		11	4
33	1593	051374	213927	0	20	20	20	0	DIR	E43	11	4
34	1594	051374	213954	0	13440	101	2240	1	M	SPAR	25	453
35	2074	051374	213956	0	606	101	606	-1	NFM		0	1
36	2096	051374	213957	0	15	1	15	4	CASE	@@@@	0	1
37	2097	051374	213958	0	606	101	606	-1	NFM		0	2
38	2114	051374	213959	0	15	1	15	4	CASE	@@@@	0	2
39	2120	051374	213959	0	0	0	0	0	LDIR		1	2
40	2120	051374	214008	0	13440	101	2240	1	K	SPAR	25	453
41	2609	051374	214023	0	31360	101	3136	1	INV	K	1	1137
42	3729	051374	214026	0	8	8	8	-1	VIBR	EVAL	1	0
43	3730	051374	214027	0	4848	101	606	-1	VIBR	U	1	0
44	3906	051374	214215	0	28224	101	3136	1	INV	K	2	1137
45	4914	051374	214218	0	8	8	8	-1	VIBR	EVAL	2	0
46	4915	051374	214219	0	4848	101	606	-1	VIBR	U	2	0
47	5091	051374	214359	0	606	101	606	1	SSOL	U	0	1
48	5113	051374	214416	0	13440	101	2240	1	KG	SPAR	25	453
49	5593	051374	214518	0	6	6	6	-1	STAB	EVAL	1	0
50	5594	051374	214519	0	3636	101	606	-1	STAB	U	1	0
51	5726	051374	214649	0	13440	101	2240	1	KMKG	SPAR	25	453
52	6206	051374	214713	0	31360	101	3136	1	INV	KMKG	1	1137
53	7326	051374	214723	1	606	101	606	1	SSOL	U	0	2

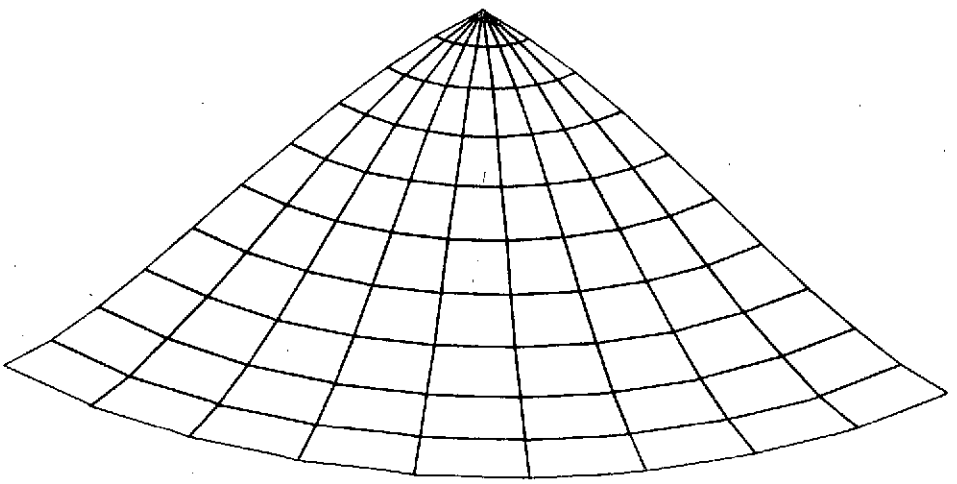
UNDEFORMED STRUCTURE



CIRCULAR PLATE

0 2
SCALE

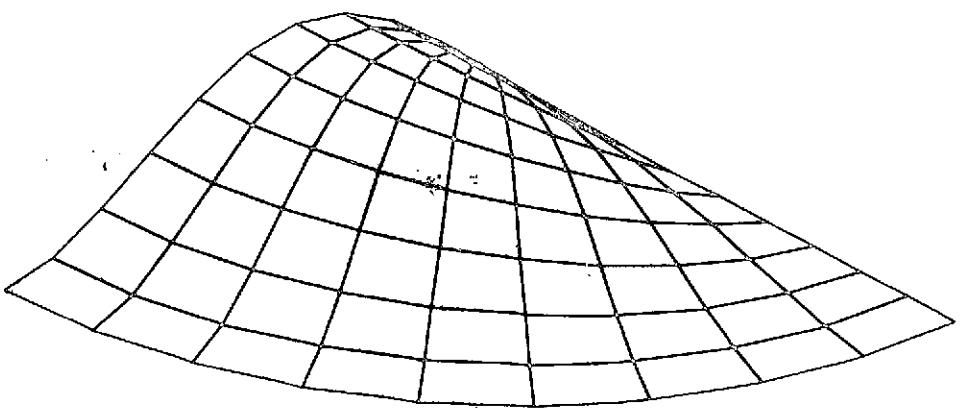
SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = .856945X10+01



CIRCULAR PLATE

0
SCALE

SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = .178274X10+02



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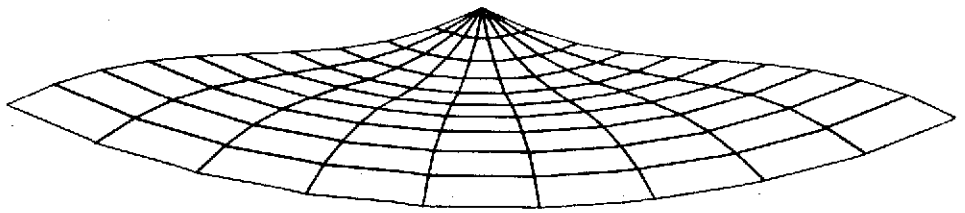
CIRCULAR PLATE

0 3
SCALE

SEQ 1 BUCKLING MODE. CRITICAL LOAD = .402798X10+03

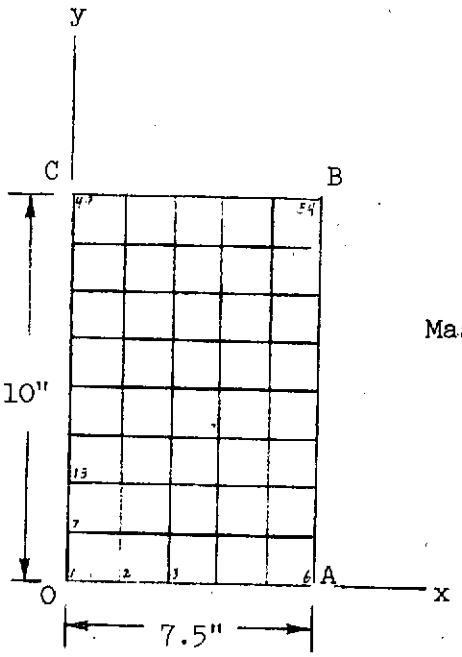


CIRCULAR PLATE

0 3
SCALE

4. RECTANGULAR PLATE

The 5 by 8 rectangular mesh shown on Figure 4-1 was used to solve the problems described in Sections 4.1-4.9.



$E = .3 \times 10^{8}$ lb/in^2

$\nu = .3$

$\alpha = .1 \times 10^{-4}$ in degrees

Mass density = .1 lb-sec^2/in^4

Thickness = .1 in

Fig. 4-1 Rectangular Plate

4.1 Vibrational Characteristics. Edges AB and BC are simply supported; and OA and OC are in symmetry planes. Results are compared with Reference 2, p 443, Eq. (188)

	ω_{11}^2	ω_{13}^2	ω_{31}^2
SPAR	$.12906 \times 10^4$	$.19435 \times 10^5$	$.48406 \times 10^5$
Ref. 2	$.12906 \times 10^4$	$.19428 \times 10^5$	$.48336 \times 10^5$

4.2 Buckling due to Unidirectional Compression. OA and OC are in symmetry planes. With AB free, a unit compressive edge load, $N_y = -1$, is applied along BC. AB and BC are then simply supported, and the critical value of N_y computed. Results are compared with Reference 3, pp 351-353

	N_y (Critical)
SPAR	$.52306 \times 10^3$
Ref. 3	$.52304 \times 10^3$

4.3 Effect of Prestress on Buckling Load. OA and OC are in symmetry planes. With edge BC free, a tensile load, $N_x = 2000$, is applied uniformly along AB. Effects on lateral stiffness of this constant load are included in the subsequent analysis. A unit edge load, $N_y = -1$, is applied along BC. AB and BC are then simply supported and N_y (critical) is computed. Results are compared with Reference 3, pp 356-358.

	N_y (Critical)
SPAR	$.12704 \times 10^4$
Ref. 3	$.12700 \times 10^4$

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4.4 Buckling due to Uniform Compression. OA and OC are in symmetry planes. Uniform compression, $N = N_x = N_y = -1$, is applied along AB and BC. AB and BC are then simply supported and N (critical) is computed. Results are compared with Reference 3, pp 356-358.

	N (Critical)
SPAR	$.18830 \times 10^3$
Ref. 3	$.18829 \times 10^3$

4.5 Effects of Prestress on Vibrational Characteristics. OA and OC are in symmetry planes. The plate is uniformly pre-tensioned by applying $N_x = N_y = 1000$, along AB and BC. With AB and BC simply supported, vibrational modes

were then computed. Results are compared with Reference 2, p 435, 443.

	ω_{11}^2	ω_{13}^2	ω_{31}^2
SPAR	$.81445 \times 10^4$	$.46029 \times 10^5$	$.90354 \times 10^5$
Ref. 2	$.81444 \times 10^4$	$.46022 \times 10^5$	$.90282 \times 10^5$

4.6 Buckling due to Linearly Varying Edge Load. OC is in a symmetry plane. With horizontal edge OA and BC free, edge AB is loaded with N_x varying linearly from N = 1 at A to N = -1 at B. Edges OA, AB, and BC are then simply supported and N (critical) computed. Results are compared with Reference 3, pp 373-379.

	N (Critical)
SPAR	$.69434 \times 10^4$
Ref. 3	$.65346 \times 10^4$

4.7 Buckling due to Edge Shear. A constant unit shear load, N_{xy}, is applied to the edges. All edges are then simply supported, and the critical shear load computed. Results are compared with Reference 3, pp 379-393.

	N_{xy} (Critical)
SPAR	$.36190 \times 10^4$
Ref. 3	$.36152 \times 10^4$

4.8 Buckling due to Thermal Load. OA and OC are in symmetry planes. Edges AB and BC are simply supported. The critical temperature, T (critical), producing lateral buckling is computed. Results are compared with Reference 3, pp 356-360, and Reference 10, Chapter 14.

	T (Critical)
SPAR	.43937 x 10
Ref. 3	.43935 x 10

4.9 Effect of Thermal Prestress on Vibrational Characteristics. OA and OC are in symmetry planes. AB and BC are simply supported. Effects of a thermal load, T = 2, are included in computing vibrational modes. Results are compared with Reference 2, p 435, p 443.

	ω_{11}^2	ω_{13}^2	ω_{31}^2
SPAR	$.70315 \times 10^3$	$.17156 \times 10^5$	$.44810 \times 10^5$
Ref. 2	$.70307 \times 10^3$	$.17149 \times 10^5$	$.44741 \times 10^5$

```
@XQT TAB                                    . GENERATE BASIC TABLES DEFINING STRUCTURE
START 54,6$
TITLE'RECTANGULAR PLATE PROBLEMS$
TEXT$
!      THE FOLLOWING PLATE PROBLEMS ARE SOLVED IN THIS RUN:
!       1 FREE VIBRATION OF A RECTANGULAR PLATE.
!       2 BUCKLING OF A RECTANGULAR PLATE, COMPRESSED IN ONE DIRECTION.
!       3 BUCKLING OF A PRE-STRESSED RECTANGULAR PLATE.
!       4 BUCKLING OF A RECTANGULAR PLATE, UNIFORM COMPRESSION.
!       5 FREE VIBRATION OF A PRE-STRESSED RECTANGULAR PLATE.
!       6 BUCKLING OF A RECTANGULAR PLATE, BENDING LOAD.
!       7 BUCKLING OF A RECTANGULAR PLATE, SHEAR LOAD.
!       8 BUCKLING OF A RECTANGULAR PLATE, TEMPERATURE LOAD.
!       9 FREE VIBRATION OF A HEATED RECTANGULAR PLATE.
!
MATERIAL CONSTANTS
     1   .3+8   .3   .1   .1-4$
JOINT LOCATIONS
     1 0.0 0.0 0.0  7.5 0.0 0.0  6 1 9
     6 0.0 10. 0.0  7.5 10. 0.0
SHELL SECTION PROPERTIES
     1 0.1$
CONSTRAINT CASE 1$
$                                           SYMMETRY AT BOUNDARY X=0.,Y=0. AND SIMPLY
$                                           SUPPORTED AT BOUNDARY X=7.5, Y=10.
     SYMMETRY PLANE=1
     SYMMETRY PLANE=2
     ZERO  3,4:      6,48,6$
     ZERO  3,5:     49,53$
     ZERO  3:       54$
CONSTRAINT CASE 2$
$                                           SYMMETRY AT X=0., SIMPLY SUPPORTED AT
$                                           Y=0., X=7.5 AND Y=10.
     SYMMETRY PLANE=1
     ZERO 2:        25$
     ZERO 3:         1,6:  12,54,6:  49,53$
     ZERO 4:        12,48,6$
     ZERO 5:         1,5:  49 53$
CONSTRAINT CASE 3$
$                                           SIMPLY SUPPORTED AT ALL BOUNDARIES.
     ZERO 1:         1,6$
     ZERO 2:         1,49,6$
     ZERO 3:         1,6:  12,54,6:  7,49,6:  50,53$
     ZERO 4:        12,48,6:  7,43,6$
     ZERO 5:         2,5:  50 53$
CONSTRAINT CASE 4$
$                                           SAME AS CASE 1, ADDITIONALLY, MOTIONS
$                                           IN X- AND Y-DIRECTION ARE CONSTRAINED AT
$                                           THE BOUNDARY X=7.5 AND Y=10.
     SYMMETRY PLANE=1
     SYMMETRY PLANE=2
     ZERO 1,2,3,4:  6,48,6$
     ZERO 1,2,3,5: 49,53$
     ZERO 1,2,3:    54$
@XQT ELD                                    . READ ELEMENT DEFINITIONS
E43 $
     1 2 8 7  1 5 8
@XQT TOPO                                   . ANALYZE ELEMENT INTERCONNECTIVITY
$
@XQT E                                      . FORM ELEMENT DATA PACKETS
$
```


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```
@XQT EKS                                    . INSERT K. S INTO ELEMENT DATA PACKETS
$
@XQT M                                      . FORM SYSTEM M
$
@XQT K                                      . FORM SYSTEM K
$
@XQT INV                                    . FACTOR K
$
@XQT EIG                                    . SOLVE SYSTEM EIGENPROBLEM
$                                             FIND FREQUENCIES FOR FREE VIBRATION OF
$                                             THE SIMPLY SUPPORTED RECTANGULAR PLATE.
      RESET INIT=6,MAXIT=1$
@XQT Q                                      . DEFINE APPLIED LOADS
      CASE 1: COMPRESSIVE FORCES IN Y-DIRECTION. APPLY AT Y=10.
        NODAL FORCES. MOMENTS
          49 2 -0.75     2 5$
          50 2 -1.5      4 1$
      CASE 2: COMPRESSIVE FORCES IN X- AND Y-DIRECTION. APPLY AT Y=10. AND X=7.5
        NODAL FORCES. MOMENTS
          49 2 -0.75     2 5$
          50 2 -1.5      4 1$
           6 1 -0.625    2 48$
          12 1 -1.25     7 6$
      CASE 3: BENDING FORCES. APPLY AT X=7.5
        NODAL FORCES. MOMENTS
           6 1  0.57292$
          12 1  0.9375   3 6  -0.3125$
          36 1 -0.3125   3 6  -0.3125$
          54 1 -0.57292$
      CASE 4: SHEAR FORCES. APPLY AT X=7.5 AND Y=10.
        NODAL FORCES. MOMENTS
           6 2  0.625    2 48$
          12 2  1.25     7 6$
          49 1  0.75     2 5$
          50 1  1.5      4 1$
      CASE 5: UNIFORM TEMPERATURE LOAD. APPLY AT ALL NODES
        NODAL TEMPERATURES
           1 1.0  54 1$
      CASE 6: TENSILE FORCES IN X-DIRECTION. APPLY AT X=7.5
        NODAL FORCES. MOMENTS
           6 1  1250.0  2 48$
          12 1  2500.0  7 6$
@XQT DSOL                                   . SOLVE LOAD CASE 1
      RESET L1=1,L2=1$
@XQT KG                                     . FORM SYSTEM KG. LOAD CASE 1
$
@XQT EIG                                    . FIND CRITICAL AMPLITUDE. LOAD CASE 1
$
      RESET PROB=STAB,INIT=6,UPPER=1$
@XQT DSOL                                   . SOLVE LOAD CASE 6
      RESET L1=6,L2=6$
@XQT KG                                     . FORM SYSTEM KG. LOAD CASE 6
      RESET QSEQ=6$
@XQT LCM
$                                           . ADD KG AND K TO FORM STIFFNESS MATRIX FOR
$                                             THE PRESTRESSED PLATE.
@XQT INV                                    . FACTOR K+KG
      RESET K=K+KG$
@XQT DSOL
$                                           . SOLVE LOAD CASE 1 FOR THE PRESTRESSED
$                                             PLATE.
```

```
      RESET K=K+KG,L1=1,L2=1$
*XQT KG
$                                          FORM GEOMETRICAL STIFFNESS MATRIX FROM
$                                          LOAD CASE 1 FOR THE PRESTRESSED PLATE.
*XQT EIG
$                                          FIND BUCKLING VALUE OF LOAD CASE 1 FOR
$                                          THE PRESTRESSED PLATE BY LOAD CASE 6
      RESET K=K+KG,PROB=STAB,INIT=4,UPPER=1$
*XQT DSOL
      RESET L1=2,L2=2$
*XQT KG                                    . FORM SYSTEM KG, LOAD CASE 2
      RESET QSEQ=2$
*XQT EIG
$                                          . FIND BUCKLING VALUE OF LOAD CASE 2.
      RESET PROB=STAB,INIT=4,UPPER=1$
*XQT LCM
$
$                                          FORM STIFFNESS MATRIX FOR A UNIFORMLY
$                                          PRESTRESSED PLATE.(NX=NY=1000)
      RESET R=KMKG,Q1=K,Q2=KG,C2=-1000.0$
*XQT INV
      RESET K=KMKG$
*XQT EIG
$
$                                          FIND FREQUENCY FOR FREE VIBRATION OF THE
$                                          UNIFORMLY PRESTRESSED PLATE.
      RESET K=KMKG,INIT=6$
*XQT INV
      RESET CON=2$
*XQT DSOL                                  . SOLVE LOAD CASE 3
      RESET CON=2,L1=3,L2=3$
*XQT KG
      RESET QSEQ=3$
*XQT EIG
$                                          FIND BUCKLING VALUE OF LOAD CASE 3.
      RESET CON=2,PROB=STAB,INIT=8,UPPER=1$
*XQT INV
      RESET CON=3$
*XQT DSOL                                  . SOLVE LOAD CASE 4
      RESET CON=3,L1=4,L2=4$
*XQT KG                                    . FORM SYSTEM KG, LOAD CASE 4
      RESET QSEQ=4$
*XQT EIG
$                                          FIND BUCKLING VALUE OF LOAD CASE 4.
      RESET CON=3,PROB=STAB,INIT=6,UPPER=1$
*XQT INV
      RESET CON=4$
*XQT DSOL                                  . SOLVE LOAD CASE 5
      RESET CON=4,L1=5,L2=5$
*XQT KG                                    . FORM SYSTEM KG, LOAD CASE 5
      RESET QSEQ=5$
*XQT EIG
$                                          FIND BUCKLING TEMPERATURE OF THE PLATE
      RESET PROB=STAB,CON=4,INIT=4,UPPER=1$
*XQT LCM
$
$                                          FORM STIFFNESS MATRIX OF THE PLATE AT
$                                          TEMPERATURE=2.
      RESET R=KTKG,Q1=K,Q2=KG,C2=2.0$
*XQT INV
      RESET K=KTKG,CON=4$
*XQT EIG
$
$                                          FIND FREQUENCIES FOR FREE VIBRATION OF
$                                          THE HEATED PLATE.
      RESET CON=4,K=KTKG,INIT=6$
*XQT DCU                                   . EXECUTE DATA COMPLEX UTILITY PROGRAM
      TOC 1$
```

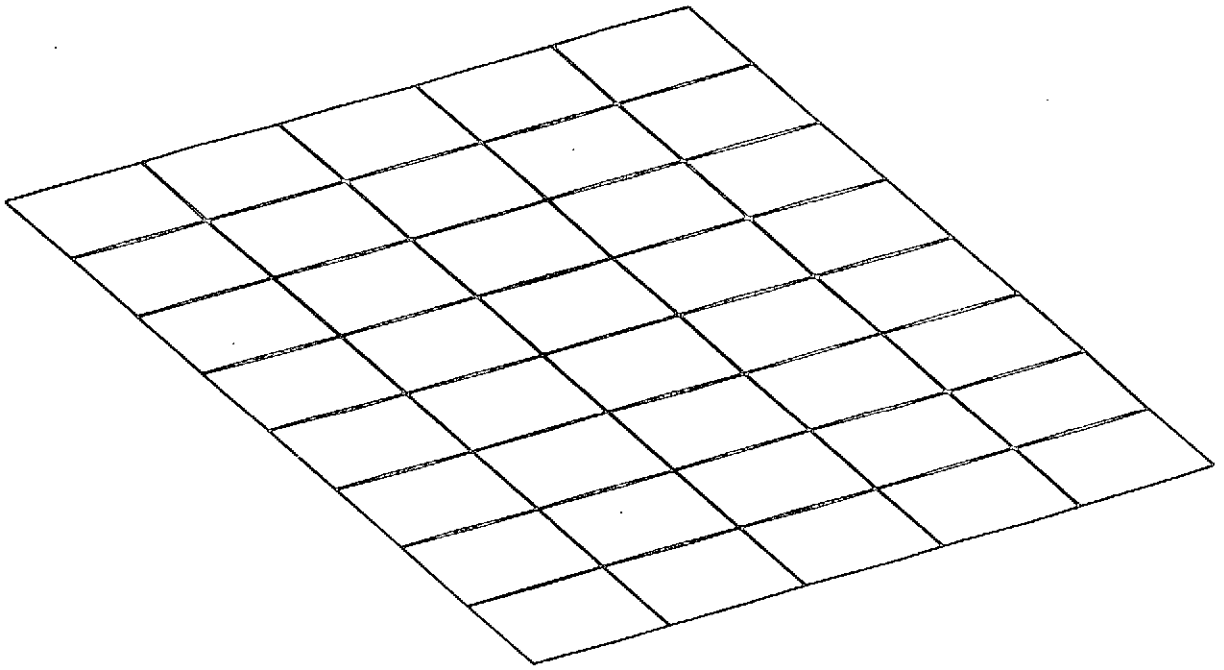
TABLE OF CONTENTS, OAL 1

RECTANGULAR PLATE PROBLEMSS

SEQ	RR	DATE	TIME	ER	WORDS	ROWS /BLK	BLK SIZE	TY	DATA SET NAME N1	N2	N3	N4
1	11	051374	172618	0	18	1	18	0	JDF1	BTAB	1	8
2	12	051374	172618	0	54	54	54	0	JDEF	BTAB	2	6
3	14	051374	172618	0	12	1	12	1	ALTR	BTAB	2	4
4	15	051374	172619	0	19	1	19	4	NDAL		0	0
5	16	051374	172619	0	200	10	200	3	TEXT	BTAB	2	1
6	24	051374	172619	0	10	1	10	1	MATC	BTAB	2	2
7	25	051374	172619	0	162	54	162	1	JLOC	BTAB	2	5
8	31	051374	172620	0	19	1	19	1	SA	BTAB	2	13
9	32	051374	172621	0	54	54	54	0	CON		1	0
10	34	051374	172621	0	54	54	54	0	CON		2	0
11	36	051374	172621	0	54	54	54	0	CON		3	0
12	38	051374	172622	0	54	54	54	0	CON		4	0
13	40	051374	172622	0	486	54	486	1	GJJT	BTAB	2	19
14	58	051374	172626	0	640	40	896	0	DEF	E43	11	4
15	90	051374	172626	0	2	1	2	0	GD	E43	11	4
16	91	051374	172626	0	15	1	15	0	GTIT	E43	11	4
17	92	051374	172626	0	12	12	12	0	NELZ	BTAB	1	11
18	93	051374	172627	0	5	1	5	0	NE		0	0
19	94	051374	172627	0	6	1	6	0	NS		0	0
20	95	051374	172627	0	1	1	1	3	ELTS	NAME	0	0
21	105	051374	172627	0	1	1	1	0	ELTS	LTYP	0	0
22	106	051374	172627	0	1	1	1	0	ELTS	NNOD	0	0
23	107	051374	172627	0	1	1	1	0	ELTS	ISCT	0	0
24	108	051374	172627	0	1	1	1	0	ELTS	NELS	0	0
25	109	051374	172627	0	1	1	1	0	ELTS	LES	0	0
26	110	051374	172630	0	1344	54	1344	0	KMAP		227	19
27	158	051374	172631	0	2688	54	1344	0	AMAP	0000	387	36
28	254	051374	172640	0	12320	40	308	4	E43		11	4
29	694	051374	172634	0	20	20	20	0	DIR	E43	11	4
30	695	051374	172648	0	6720	54	2240	1	M	SPAR	25	227
31	935	051374	172649	0	324	54	324	-1	NFM		0	1
32	947	051374	172650	0	15	1	15	4	CASE	0000	0	1
33	948	051374	172650	0	324	54	324	-1	NFM		0	2
34	960	051374	172651	0	15	1	15	4	CASE	0000	0	2
35	961	051374	172651	0	324	54	324	-1	NFM		0	3
36	973	051374	172651	0	15	1	15	4	CASE	0000	0	3
37	974	051374	172652	0	324	54	324	-1	NFM		0	4
38	986	051374	172652	0	15	1	15	4	CASE	0000	0	4
39	987	051374	172652	0	54	54	54	-1	NYMP		0	5
40	989	051374	172653	0	324	54	324	-1	NFM		0	5
41	1010	051374	172654	0	15	1	15	4	CASE	0000	0	5
42	1011	051374	172654	0	324	54	324	-1	NFM		0	6
43	1023	051374	172654	0	15	1	15	4	CASE	0000	0	6
44	1024	051374	172655	0	0	0	0	0	LDIR		1	6
45	1024	051374	172701	0	6720	54	2240	1	K	SPAR	25	227
46	1264	051374	172705	0	9408	54	3136	1	INV	K	1	387
47	-1600	051374	172707	0	6	6	6	-1	VIBR	EVAL	1	0
48	-1601	051374	172708	0	1944	54	324	-1	VIBR	U	1	0
49	-1673	051374	172754	0	324	54	324	1	SSOL	U	0	1
50	-1685	051374	172802	0	6720	54	2240	1	KG	SPAR	25	227
51	-1925	051374	172804	0	6	6	6	-1	STAB	EVAL	1	0
52	-1926	051374	172804	0	1944	54	324	-1	STAB	U	1	0

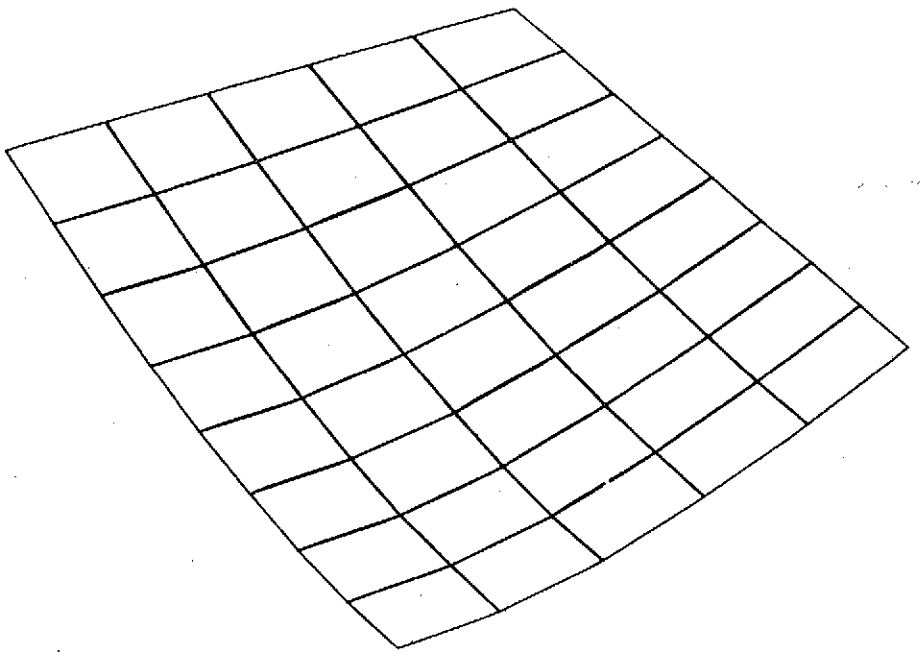
53	1998	051374	172842	0	324	54	324	1	SSOL	U	0	6	
54	-2010	051374	172850	0	6720	54	2240	1	KG	SPAR	25	227	
55	2250	051374	172851	0	6720	54	2240	1	K+KG	SPAR	25	227	
56	2490	051374	172858	0	9408	54	3136	1	INV	K+KG	1	387	
57	2826	051374	172902	0	324	54	324	1	SSOL	U	0	1	
58	-2838	051374	172910	0	6720	54	2240	1	KG	SPAR	25	227	
59	-3078	051374	172912	0	4	4	4	-1	STAB	EVAL	1	0	
60	-3079	051374	172912	0	1296	54	324	-1	STAB	U	1	0	
61	3136	051374	172950	0	324	54	324	1	SSOL	U	0	2	
62	-3148	051374	173000	0	6720	54	2240	1	KG	SPAR	25	227	
63	3388	051374	173003	0	4	4	4	-1	STAB	EVAL	1	0	
64	3389	051374	173003	0	1296	54	324	-1	STAB	U	1	0	
65	3437	051374	173037	0	6720	54	2240	1	KMKG	SPAR	25	227	
66	3677	051374	173048	0	9408	54	3136	1	INV	KMKG	1	387	
67	4013	051374	173053	0	6	6	6	-1	VIBR	EVAL	1	0	
68	4014	051374	173054	0	1944	54	324	-1	VIBR	U	1	0	
69	4086	051374	173231	0	9408	54	3136	1	INV	K	2	387	
70	4422	051374	173245	0	324	54	324	1	SSOL	U	0	3	
71	-4434	051374	173254	0	6720	54	2240	1	KG	SPAR	25	227	
72	4674	051374	173257	0	8	8	8	-1	STAB	EVAL	2	0	
73	4675	051374	173258	0	2592	54	324	-1	STAB	U	2	0	
74	4771	051374	173420	0	9408	54	3136	1	INV	K	3	387	
75	5107	051374	173425	0	324	54	324	1	SSOL	U	0	4	
76	-5119	051374	173434	0	6720	54	2240	1	KG	SPAR	25	227	
77	5359	051374	173501	0	6	6	6	-1	STAB	EVAL	3	0	
78	5360	051374	173502	0	1944	54	324	-1	STAB	U	3	0	
79	5432	051374	173722	0	9408	54	3136	1	INV	K	4	387	
80	5768	051374	173753	0	324	54	324	1	SSOL	U	0	5	
81	5789	051374	173815	0	6720	54	2240	1	KG	SPAR	25	227	
82	6029	051374	173819	0	4	4	4	-1	STAB	EVAL	4	0	
83	6030	051374	173821	0	1296	54	324	-1	STAB	U	4	0	
84	6078	051374	173934	0	6720	54	2240	1	KTKG	SPAR	25	227	
85	6318	051374	174003	0	9408	54	3136	1	INV	KTKG	4	387	
86	6654	051374	174020	0	6	6	6	-1	VIBR	EVAL	4	0	
87	6655	051374	174021	0	1944	54	324	-1	VIBR	U	4	0	

UNDEFORMED STRUCTURE



RECTANGULAR PLATE

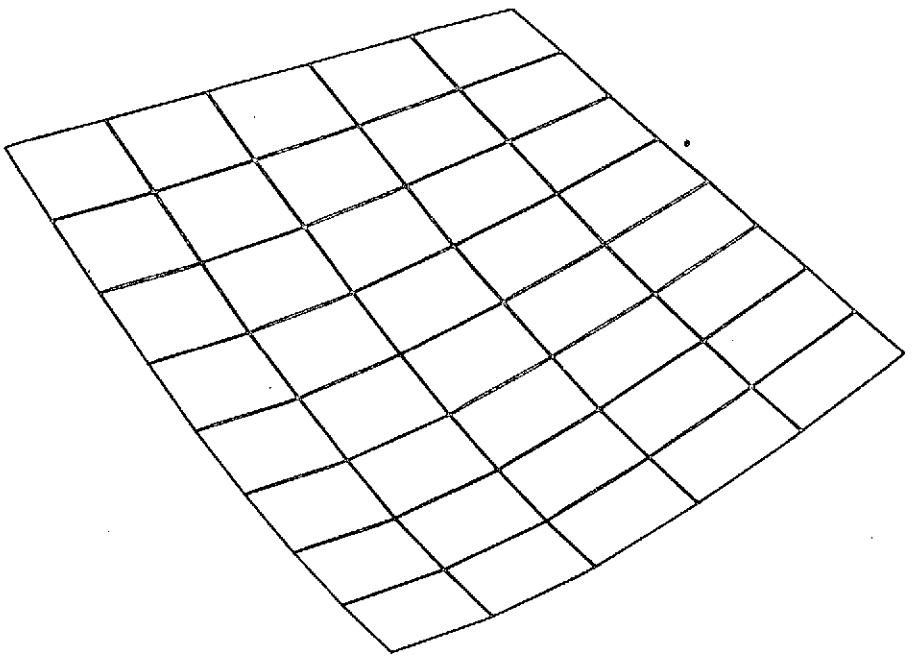
SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = .571766X10^{+01}



RECTANGULAR PLATE

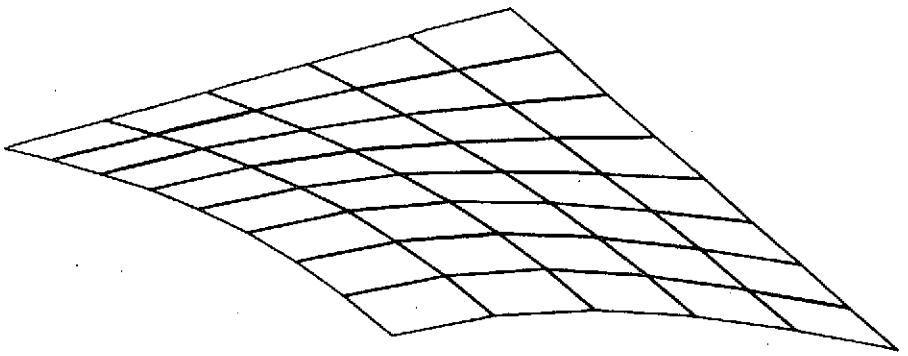
0 2
SCALE

SEQ 1 BUCKLING MODE. CRITICAL LOAD = .523060X10**03



RECTANGULAR PLATE

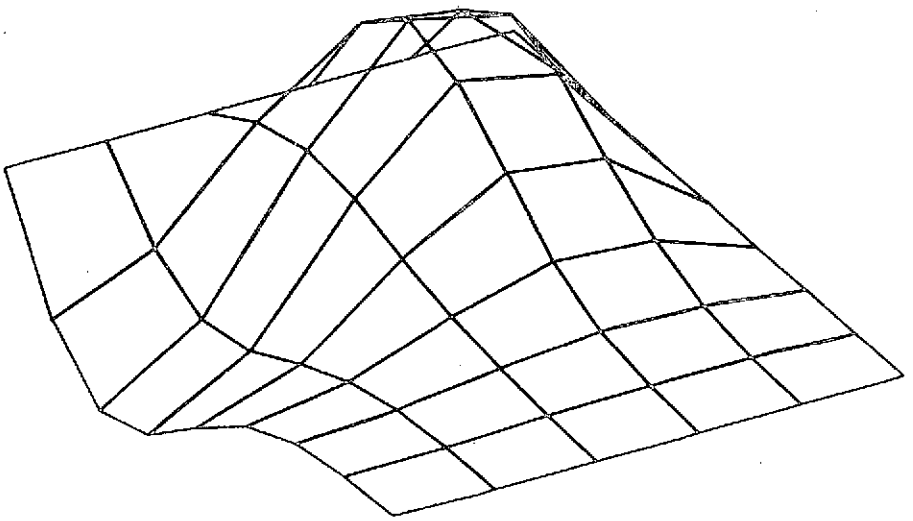
SEQ 1 BUCKLING MODE. CRITICAL LOAD = .188302X10^{+03}



RECTANGULAR PLATE

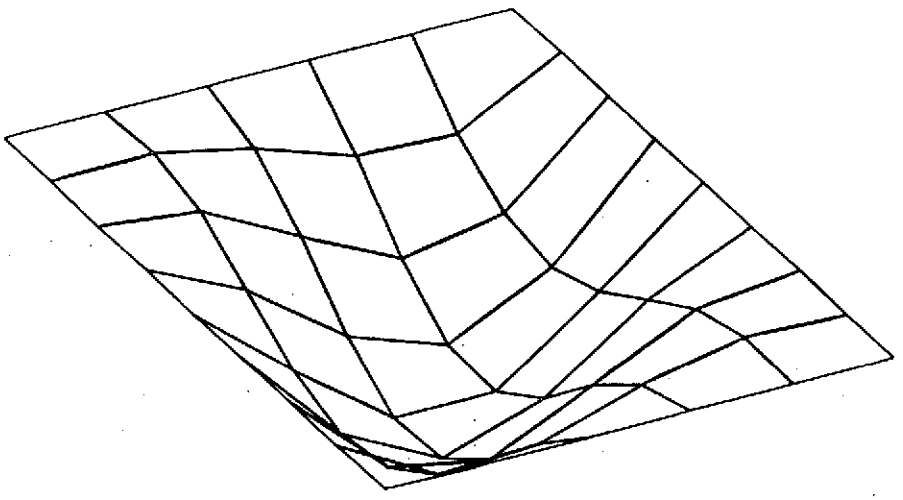
0 SCALE 2

SEQ 1 BUCKLING MODE. CRITICAL LOAD = .694336X10**04



RECTANGULAR PLATE

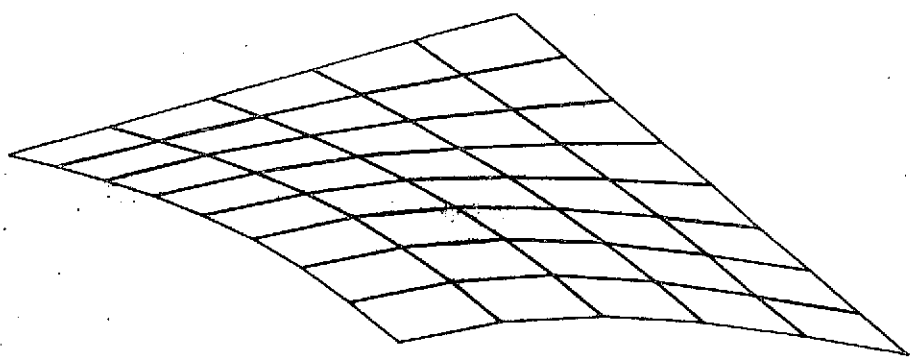
SEQ 1 BUCKLING MODE. CRITICAL LOAD = .361898X10+04



RECTANGULAR PLATE

0 2
SCALE

SEQ 1 BUCKLING MODE, CRITICAL LOAD = .439371X10**01

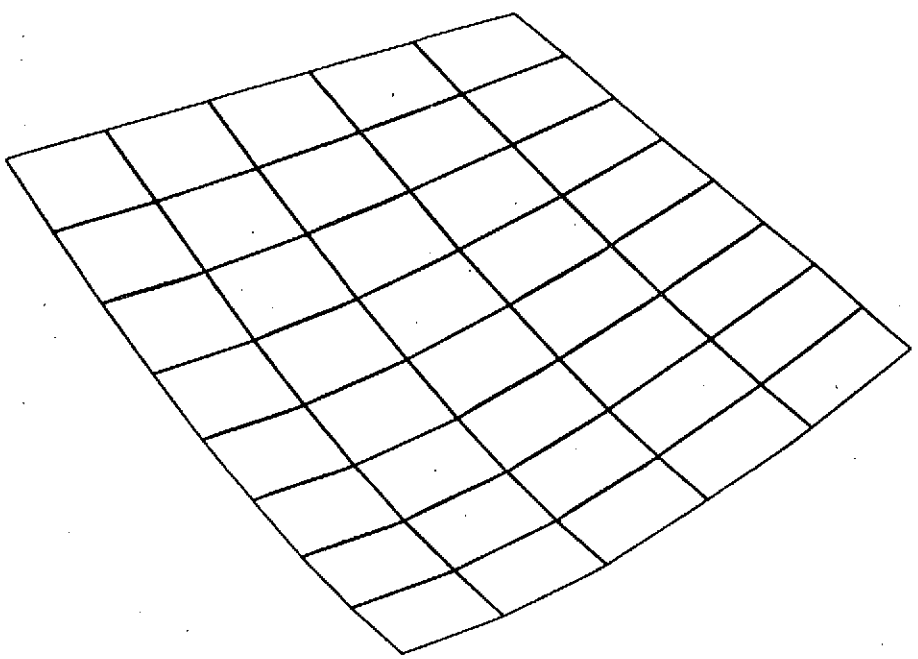


REPRODUCIBILITY OF THE
ORIGINAL PAGE IS POOR

RECTANGULAR PLATE

0 2
SCALE

SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = .422032X10^{+01}

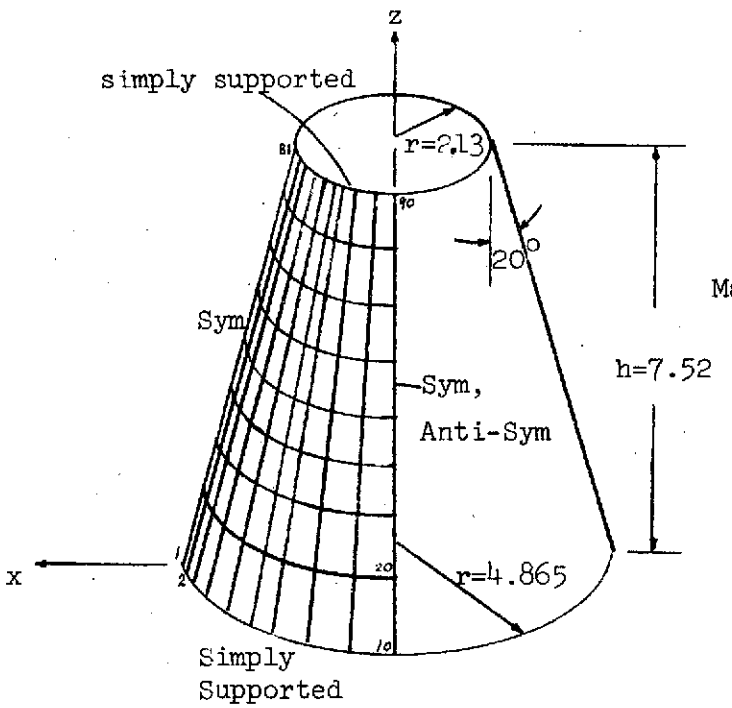


RECTANGULAR PLATE

0
SCALE

5. VIBRATION OF A CONICAL SHELL

Vibrational modes of a simply supported truncated conical shell were obtained. One quarter of the shell was used for the finite element model. The quarter shell was divided into a 9 x 8 finite element network. The seven lowest frequencies were obtained. These seven frequencies correspond to half waves in the meridional direction with wave numbers three to nine in the circumferential direction. The geometrical and material properties of the shell are indicated in the figure below:



$E = .3 \times 10^8$ lb /in^2

$\nu = .3$

Mass density = .1 lb-sec^2/in^4

t = .02 in

Results are compared with Reference 11:

Circumferential Wave No. n	SPAR	ω_{on}^2 Seide
3	$.19244 \times 10^6$	$.22078 \times 10^6$
4	$.80675 \times 10^5$	$.89014 \times 10^5$
5	$.52175 \times 10^5$	$.56966 \times 10^5$
6	$.55235 \times 10^5$	$.59480 \times 10^5$
7	$.73549 \times 10^5$	$.77386 \times 10^5$
8	$.99957 \times 10^5$	$.10340 \times 10^6$
9	$.13408 \times 10^6$	$.13724 \times 10^6$

Since Reference 11 neglected longitudinal inertia terms, the resulting frequency values are higher than the exact solution. For low values of circumferential wave number, most of the energy of the shell is associated with membrane action. Therefore, neglecting the longitudinal inertia terms results in significant error.

```
@XQT TAB                                    . GENERATE BASIC TABLES DEFINING STRUCTURE
START 90$
TITLE VIBRATION OF A TRUNCATED CONICAL SHELL
MATERIAL CONSTANTS$
      1 .3+8 .3 .3 1.-5 1.+4$
ALTERNATE REFERENCE FRAMES$                 DEFINE REFERENCE FRAMES PARALLEL TO
$                                           MERIDIONAL, RADIAL, CIRCUMFERENTIAL.
$                                           COORDINATE LINES OF THE CONE.
       2 3  0. 2 -20.$
       3 3 10. 2 -20.$
       4 3 20. 2 -20.$
       5 3 30. 2 -20.$
       6 3 40. 2 -20.$
       7 3 50. 2 -20.$
       8 3 60. 2 -20.$
       9 3 70. 2 -20.$
      10 3 80. 2 -20.$
      11 3 90. 2 -20.$
JOINT REFERENCE FRAMES$
      NREF= 2: 1,81,10$
      NREF= 3: 2,82,10$
      NREF= 4: 3,83,10$
      NREF= 5: 4,84,10$
      NREF= 6: 5,85,10$
      NREF= 7: 6,86,10$
      NREF= 8: 7,87,10$
      NREF= 9: 8,88,10$
      NREF=10: 9,89,10$
      NREF=11:10,90,10$
JOINT LOCATIONS$
      FORMAT=2$
       1 2.130 0. 7.52 2.130 90. 7.52 10 1 9$
      10 4.860 0. 0.00 4.865 90. 0.00$
SHELL SECTION PROPERTIES$
      1 .02$
CONSTRAINT CASE 1$
$                                           SIMPLY SUPPORTED AT BOTH ENDS OF THE CONE
$                                           SYMMETRY AT THETA=0 PLANE, ANTI-SYMMETRY
$                                           AT THETA=90 DEGREE PLANE.
      ANTISYMMTRY PLANE=1$
      SYMMETRY PLANE=2$
      ZERO 1,2: 1,10: 81,90$
CONSTRAINT CASE 2$
$                                           SIMPLY SUPPORTED AT BOTH ENDS OF THE CONE
$                                           SYMMETRY AT BOTH THETA=0 AND 90 DEGREE
$                                           PLANES.
      SYMMETRY PLANE=1$
      SYMMETRY PLANE=2$
      ZERO 1,2: 1,10: 81,90$
@XQT ELD                                    . READ ELEMENT DEFINITIONS
E43 $
      1 2 12 11  1 9 8$
@XQT E                                      . FORM ELEMENT DATA PACKETS
@XQT EKS                                    . INSERT K, S INTO ELEMENT DATA PACKETS
@XQT K                                      . FORM SYSTEM K
@XQT INV                                    . FACTOR MATRIX IN K SPAR FORMAT
@XQT M                                      . FORM SYSTEM M
@XQT EIG                                    . SOLVE SYSTEM EIGENPROBLEM
$                                             FIND FREQUENCIS AND CORRESPONDING MODE
$                                             SHAPE FOR FREE VIBRATION OF THE CONE
```


REPRODUCIBILITY OF THE
ORIGINAL PAGE IS POOR

```
$                                          UNDER CONSTRAINT CONDITION 1.
      RESET INIT=12$
      PRINT 0 0 0 0 1$
@XQT INV
      RESET CON=2$
@XQT EIG
$                                          FIND FREQUENCIS AND CORRESPONDING MODE
$                                          SHAPE FOR FREE VIBRATION OF THE CONE
$                                          UNDER CONSTRAINT CONDITION 2.
      RESET INIT=12,CON=2$
      PRINT 0 0 0 0 1$
@XQT OCU                                   EXECUTE DATA COMPLEX UTILITY PROGRAM
      TOC 1$
```

TABLE OF CONTENTS, DAL 1

VIBRATION OF A TRUNCATED CONICAL SHELL

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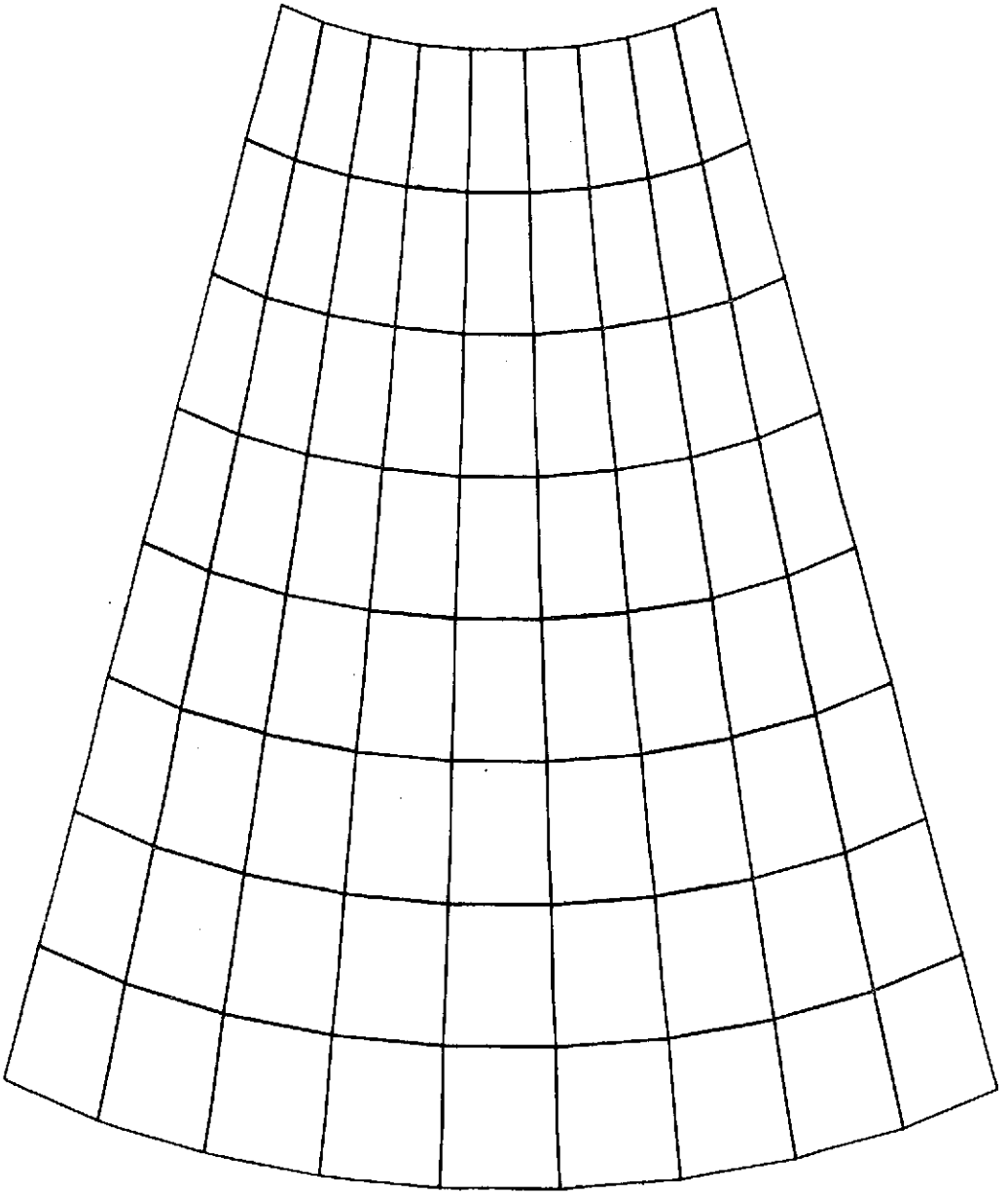
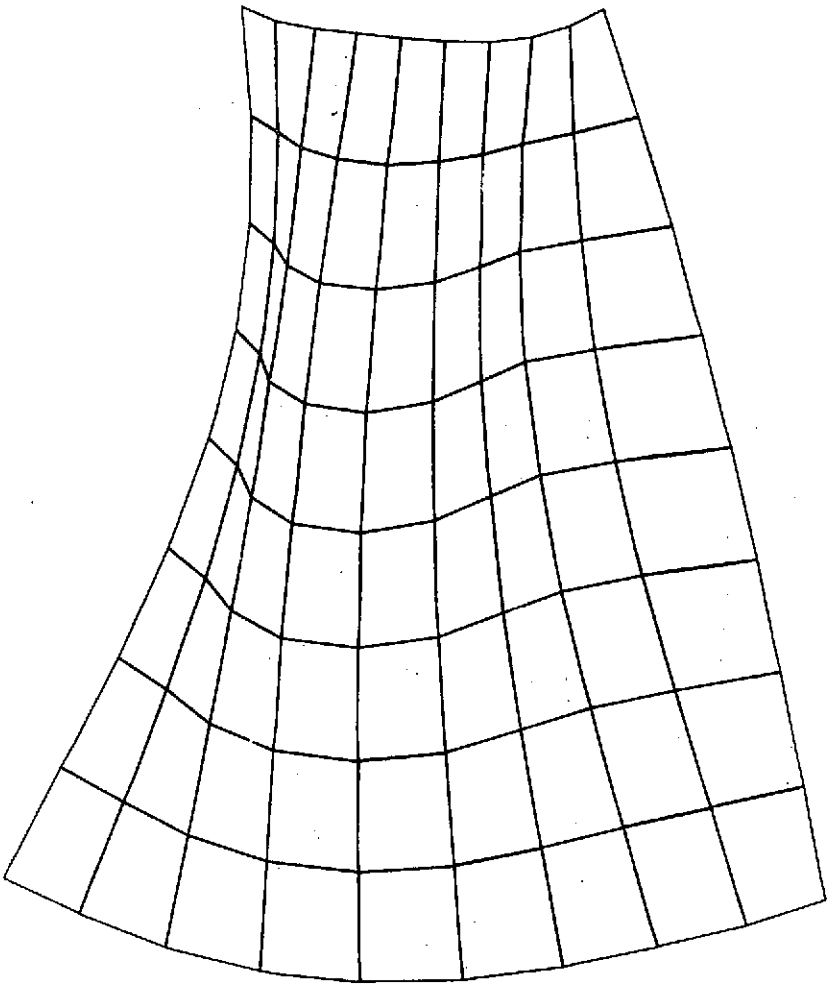
3	-16	041874	131339	0	12	1	12	1	ALTR	BTAB	2	4
4	17	041874	131339	0	19	1	19	4	NDAL		0	0
5	18	041874	131339	0	10	1	10	1	MATC	BTAB	2	2
6	19	041874	131340	0	132	11	132	1	ALTR	BTAB	2	4
7	24	041874	131340	0	90	90	90	0	JREF	BTAB	2	6
8	28	041874	131340	0	270	90	270	1	JLOC	BTAB	2	5
9	38	041874	131340	0	19	1	19	1	SA	BTAB	2	13
10	39	041874	131342	0	90	90	90	0	CON		1	0
11	43	041874	131342	0	90	90	90	0	CON		2	0
12	47	041874	131342	0	810	90	810	1	QJJT	BTAB	2	19
13	76	041874	131418	0	1152	72	896	0	DEF	E43	11	4
14	140	041874	131418	0	2	1	2	0	GD	E43	11	4
15	141	041874	131418	0	15	1	15	0	GTIT	E43	11	4
16	142	041874	131418	0	12	12	12	0	NELZ	BTAB	1	11
17	143	041874	131418	0	5	1	5	0	KE		0	0
18	144	041874	131418	0	6	1	6	0	NS		0	0
19	145	041874	131418	0	1	1	1	3	ELTS	NAME	0	0
20	146	041874	131418	0	1	1	1	0	ELTS	LTYP	0	0
21	156	041874	131418	0	1	1	1	0	ELTS	NNOD	0	0
22	157	041874	131419	0	1	1	1	0	ELTS	ISCT	0	0
23	158	041874	131419	0	1	1	1	0	ELTS	NELS	0	0
24	159	041874	131419	0	1	1	1	0	ELTS	LE3	0	0
25	160	041874	131421	0	2688	90	1344	0	KMAP		395	27
26	256	041874	131424	0	8064	90	1344	0	AMAP	●●●	971	78
27	544	041874	131433	0	22176	72	308	4	E43		11	4
28	1336	041874	131427	0	20	20	20	0	DIR	E43	11	4
29	1337	041874	131440	0	15680	90	2240	1	K	SPAR	36	395
30	1897	041874	131501	0	37632	90	3136	1	INV	K	1	971
31	3241	041874	131518	0	15680	90	2240	1	M	SPAR	36	395
32	3801	041874	131521	0	12	12	12	-1	VIBR	EVAL	1	0
33	3802	041874	131521	0	6480	90	540	-1	VIBR	U	1	0
34	4042	041874	131855	0	37632	90	3136	1	INV	K	2	971
35	5386	041874	131858	0	12	12	12	-1	VIBR	EVAL	2	0
36	5387	041874	131858	0	6480	90	540	-1	VIBR	U	2	0

UNDEFORMED STRUCTURE



TRUNCATED CONICAL SHELL

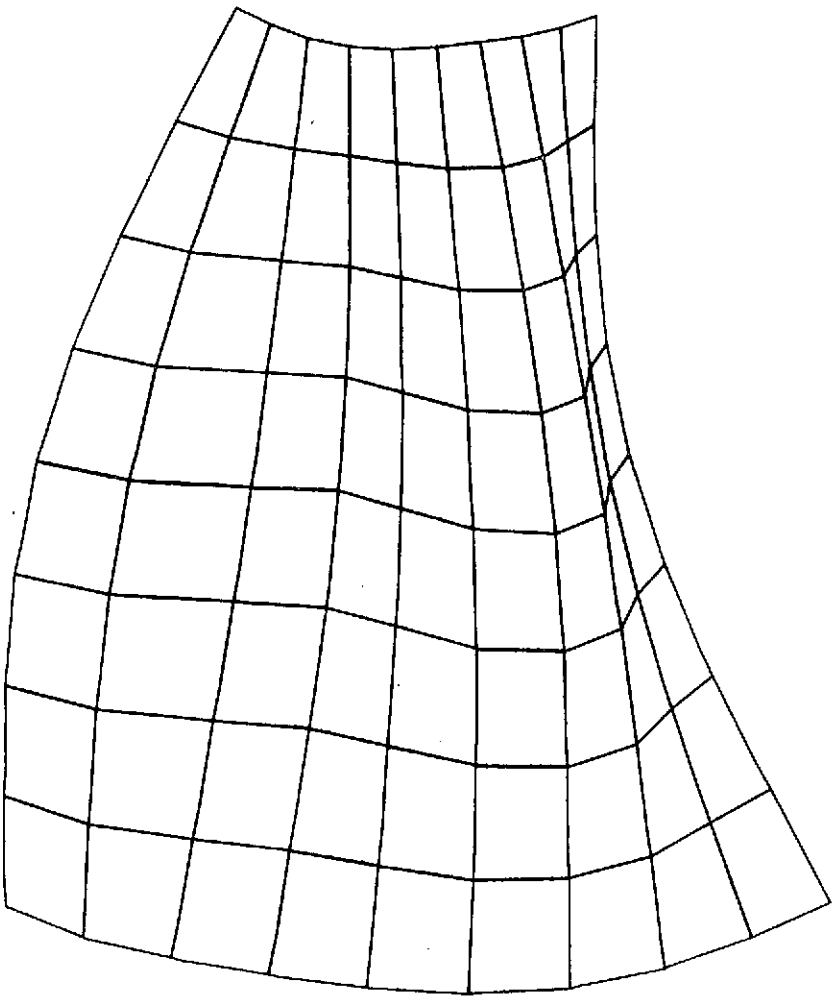
SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = .363540X10+02



TRUNCATED CONICAL SHELL

0 2
SCALE

SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = .374047X10^{+02}

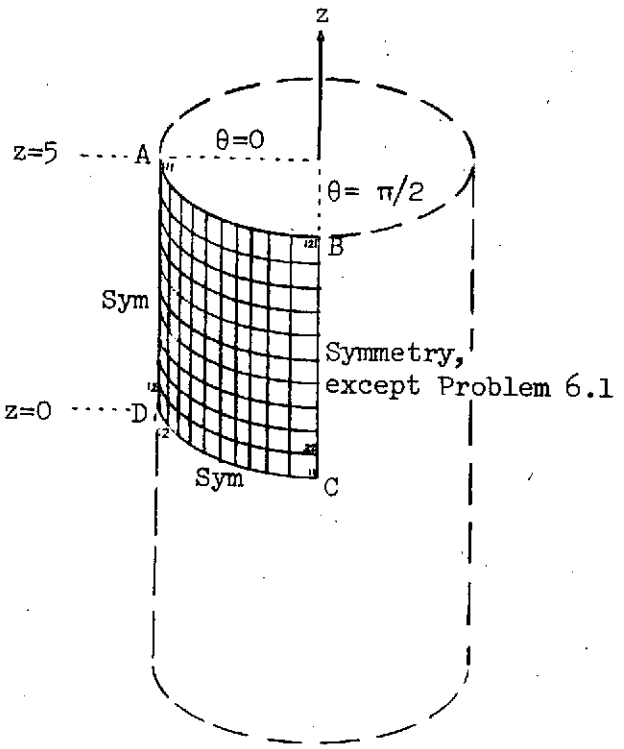


TRUNCATED CONICAL SHELL

0 2
SCALE

6. CYLINDRICAL SHELL

The 10 by 10 cylindrical mesh shown on Figure 6-1 was used to solve the problems discussed in Sections 6.1 through 6.5.



$E = .1 \times 10^8$ lb/in^2

$\nu = .3$

Mass density = .1 lb-sec^2/in^4

Thickness = .1 in

Radius = 10. in

Fig. 6-1 Cylindrical Shell

6.1 Vibrational Characteristics. With edge AB simply supported, vibrational modes were computed with (1) BC in a symmetry plane, and (2) BC in an anti-symmetry plane. Results are compared with Reference 6, p 2154.

Circumferential Wave No., n	ω_{on}^2 SPAR	Reference 6
4	$.1457 \times 10^6$	$.1425 \times 10^6$
5	$.8870 \times 10^5$	$.8863 \times 10^5$
6	$.6393 \times 10^5$	$.6330 \times 10^5$
7	$.5828 \times 10^5$	$.5813 \times 10^5$
8	$.6608 \times 10^5$	$.6596 \times 10^5$
9	$.8548 \times 10^5$	$.8493 \times 10^5$
10	$.1168 \times 10^6$	$.1125 \times 10^6$
11	$.1609 \times 10^6$	$.1587 \times 10^6$

6.2 Buckling due to Axial Compression. A unit compressive edge load, N_z, is applied to AB. AB is then simply supported and critical value of N_z computed. Results are compared with Reference 3, pp 457-458.

	N_z (Critical)
SPAR	$.6068 \times 10^4$
Ref. 3	$.6075 \times 10^4$

6.3 Effects of Pretensioning on Buckling due to External Pressure. The shell is pretensioned by a line load, $N_z = 2200$, along AB. AB is then simply supported, and the critical external pressure, P_c, is computed. Results are compared with Reference 5, p 425, Figure 9.

	P_c
SPAR	$.1415 \times 10^3$
Ref. 5	$.1406 \times 10^3$

6.4 Buckling due to External Pressure. Edge AB is simply supported. The critical pressure, P_c, is computed. Results are compared with Reference 5,

p 433, Figure 15.

	P_c
SPAR	$.1084 \times 10^3$
Ref. 5	$.1045 \times 10^3$

6.5 Effects of Pressurization on Buckling due to Axial Compression. With edge AB free, an internal pressure, P = 220, is applied. The stiffening effect of this constant preload is included in the subsequent analysis. A unit compressive load, N_z, is applied along AB. AB is then simply supported, and N_z (critical) computed. Results are compared with Reference 4, p 425, Figure 9.

	N_z (Critical)
SPAR	$.6082 \times 10^4$
Ref. 4	$.6150 \times 10^4$

```
@XQT TAB                                . GENERATE BASIC TABLES DEFINING STRUCTURE
START 121$
TITLE1 CYLINDRICAL SHELL PROBLEMS
TEXT$
!      THE FOLLOWING PROBLEMS IN CYLINDRICAL SHELLS ARE SOLVED IN THIS RUN:
!       1  VIBRATION OF A CYLINDRICAL SHELL
!       2  BUCKLING OF A CYLINDRICAL SHELL DUE TO UNIFORM AXIAL COMPRESSION
!       3  BUCKLING OF A AXIALLY PRE-STRESSED CYLINDER DUE TO PRESSURE LOAD.
!       4  BUCKLING OF A CYLINDRICAL SHELL DUE TO PRESSURE LOAD
!       5  BUCKLING OF A PRESSURIZED CYLINDER DUE TO AXIAL COMPRESSION
!
MATERIAL CONSTANTS
    1  .1+8  .3  .1  .1-4$
JOINT LOCATIONS$
    FORMAT=2$
      1  10.0 0.0  0.  10.0 90.0  0.  11 1 11
     11  10.0 0.0  5.  10.0 90.0  5.
JOINT REFERENCE FRAME ASSIGNMENTS$
    NREF=-1; 1,121$
SHELL SECTION PROPERTIES$
    1 0.1$
CONSTRAINT CASE 1$
$                                       SIMPLY SUPPORTED AT ONE END, SYMMETRY AT
$                                       THE OTHER END AND AT PLANES THETA=0 AND
$                                       90 DEGREES.
    SYMMETRY PLANE=3
    SYMMETRY PLANE=2
    SYMMETRY PLANE=1
     ZERO 1,21 111,121$
CONSTRAINT CASE 2$
$                                       SIMPLY SUPPORTED AT ONE END, SYMMETRY AT
$                                       THE OTHER END AND AT PLANE THATA=0, ANTI-
$                                       SYMMETRY AT THETA=90 DEGREES.
    SYMMETRY PLANE=3
    SYMMETRY PLANE=2
    ANTISYMMETRY PLANE=1
     ZERO 1,21 111,121$
CONSTRAINT CASE 3$
$                                       SYMMETRY AT PLANES AT THETA=0 AND 90
$                                       DEGREES, AND AT ONE END OF THE CYLINDER.
    SYMMETRY PLANE=3
    SYMMETRY PLANE=2
    SYMMETRY PLANE=1
@XQT ELD                                . READ ELEMENT DEFINITIONS
E43 $
    1 2 13 12  1 10 10$
@XQT TOPO                               . ANALYZE ELEMENT INTERCONNECTIVITY
$
@XQT E                                  . FORM ELEMENT DATA PACKETS
$
@XQT EKS                                . INSERT K, S INTO ELEMENT DATA PACKETS
$
@XQT K                                  . FORM SYSTEM K
$
@XQT INV                                . FACTOR MATRIX IN K SPAR FORMAT
$
@XQT M                                  . FORM SYSTEM M
$
@XQT EIG                                . SOLVE SYSTEM EIGENPROBLEM
$                                         FIND FREQUENCIES AND CORRESPONDING MODE
$                                         SHAPES FOR FREE VIBRATION OF THE CYLINDER
```

REPRODUCIBILITY OF THE
ORIGINAL PAGE IS POOR

```
$                                                    UNDER CONSTRAINT CASE 1
      RESET INIT=10$
      PRINT 0 0 0 0 1$
@XQT INV
      RESET CON=2$
@XQT EIG
$                                                    FIND FREQUENCIES AND CORRESPONDING MODE
$                                                    SHAPES FOR FREE VIBRATION OF THE CYLINDE
$                                                    UNDER CONSTRAINT CASE 2
      RESET INIT=10,CON=2$
      PRINT 0 0 0 0 1$
@XQT Q                                               * DEFINE APPLIED LOADS
      CASE 1: UNIFORM AXIAL COMPRESSIVE FORCE APPLY AT ONE END
        NODAL FORCES, MOMENTS
            111 3  -0.7854   2 10$
            112 3  -1.5708   9 1$
      CASE 2: UNIFORM RADIAL PRESSURE
        NODAL PRESSURE
           1  -1.0    121 1$
@XQT INV
      RESET CON=3$
@XQT DSOL                                            * SOLVE LOAD CASE 1
      RESET L1=1,L2=1,CON=3$
@XQT KG                                              * FORM SYSTEM KG, AXIAL COMPRESSION
@XQT EIG
$                                                    FIND BUCKLING AXIAL LOAD OF THE CYLINDER
      RESET PROBLEM=STABILITY,INIT=4,UPPER=1,NDYN=10$
@XQT LCM                                             * FORM LINEAR COMBINATION OF MATRICES
$                                                    FORM STIFFNESS MATRIX OF AN AXIALLY PRE-
$                                                    STRESSED CYLINDRICAL SHELL, AXIAL TENSILE
$                                                    FORCE=2200.
      RESET R=KXKG,Q1=K,Q2=KG,C2=-2200.$
@XQT INV
      RESET K=KXKG,CON=3$
@XQT DSOL
      RESET L1=2,L2=2,CON=3,K=KXKG$
@XQT KG
      RESET QSEQ=2$
@XQT INV
      RESET K=KXKG$
@XQT EIG
$                                                    FIND BUCKLING PRESSURE OF PRE-STRESSED
$                                                    CYLINDER
      RESET INIT=4,UPPER=1,PROB=STAB,NDYN=10,K=KXKG$
@XQT DSOL
      RESET L1=2,L2=2,CON=3$
@XQT KG                                              * FORM SYSTEM KG, UNIFORM PRESSURE
      RESET QSEQ=2$
@XQT EIG
$                                                    FIND BUCKLING PRESSURE OF THE CYLINDER
      RESET INIT=4,UPPER=1,PROB=STAB,NDYN=10$
@XQT LCM
$                                                    FORM STIFFNESS MATRIX OF A PRESSURIZED
$                                                    CYLINDER, INTERNAL PRESSURE=220
      RESET R=KYKG,Q1=K,Q2=KG,C2=-220.$
@XQT INV
      RESET K=KYKG,CON=3$
@XQT DSOL
      RESET K=KYKG,CON=3,L1=1,L2=1$
@XQT KG
@XQT INV
```

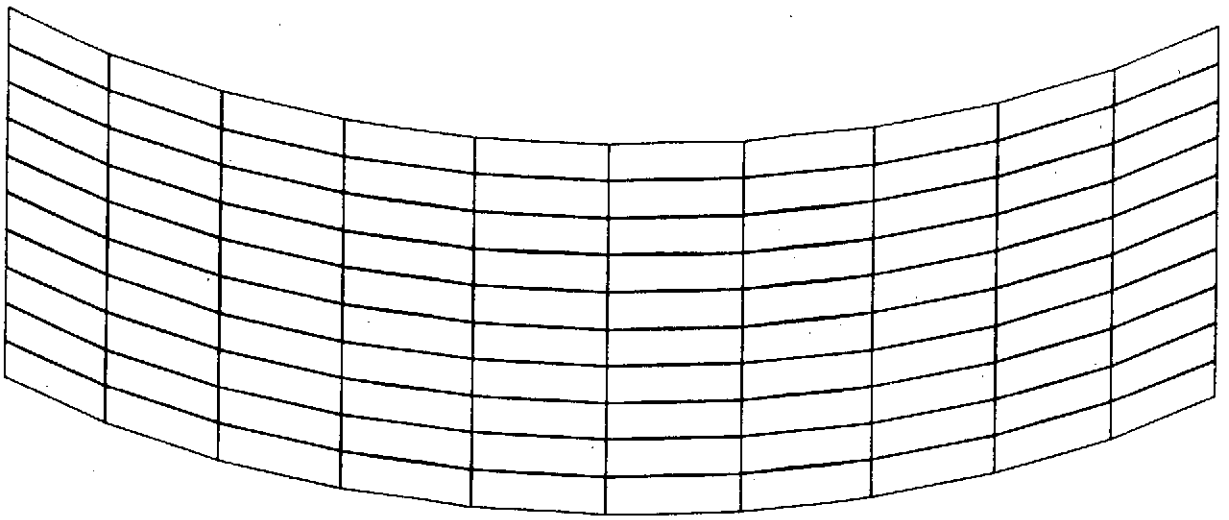
```
      RESET K=KYKG$
@XQT EIG
$ ------------------------------------------ FIND BUCKLING AXIAL LOAD OF THE PRE-
$                                            PRESSURIZED CYLINDER
      RESET INIT=4,UPPER=1,PROB=STAB,NDYN=10,K=KYKG$
@XQT DCU                                   . EXECUTE DATA COMPLEX UTILITY PROGRAM
      TOC 1$
```

CYLINDRICAL SHELL PROBLEMS

SEQ	RR	DATE	TIME	ER	WORDS	ROWS /BLK	BLK SIZE	TY	DATA NAME N1	SET NAME N2	N3	N4
1	11	061174	121857	0	18	1	18	0	JDF1	BTAB	1	8
2	-12	061174	121857	0	121	121	121	0	JREF	BTAB	2	6
3	17	061174	121857	0	12	1	12	1	ALTR	BTAB	2	4
4	18	061174	121857	0	19	1	19	4	NDAL		0	0
5	19	061174	121858	0	140	7	140	3	TEXT	BTAB	2	1
6	24	061174	121858	0	10	1	10	1	MATC	BTAB	2	2
7	25	061174	121901	0	363	121	363	1	JLOC	BTAB	2	5
8	38	061174	121901	0	121	121	121	0	JREF	BTAB	2	6
9	43	061174	121901	0	19	1	19	1	SA	BTAB	2	13
10	44	061174	121902	0	121	121	121	0	CON		1	0
11	49	061174	121903	0	121	121	121	0	CON		2	0
12	54	061174	121905	0	121	121	121	0	CON		3	0
13	59	061174	121906	0	1089	121	1089	1	QJJT	BTAB	2	14
14	98	061174	121909	0	1600	100	896	0	DEF	E43	11	4
15	162	061174	121909	0	2	1	2	0	GD	E43	11	4
16	163	061174	121910	0	15	1	15	0	GTIT	E43	11	4
17	164	061174	121910	0	12	12	12	0	NELZ	BTAB	1	11
18	165	061174	121910	0	5	1	5	0	KE		0	0
19	166	061174	121910	0	6	1	6	0	MS		0	0
20	167	061174	121910	0	1	1	1	3	ELTS	NAME	0	0
21	177	061174	121910	0	1	1	1	0	ELTS	LTYP	0	0
22	178	061174	121910	0	1	1	1	0	ELTS	NNOD	0	0
23	179	061174	121910	0	1	1	1	0	ELTS	ISCT	0	0
24	180	061174	121910	0	1	1	1	0	ELTS	NELS	0	0
25	181	061174	121911	0	1	1	1	0	ELTS	LE3	0	0
26	182	061174	121922	0	4032	121	1344	0	KMAP		541	29
27	526	061174	121925	0	12096	121	1344	0	AMAP	****	1441	91
28	758	061174	121954	0	3060	100	308	4	E43		11	4
29	1858	061174	121936	0	20	20	20	0	DIP	E43	11	4
30	1859	061174	122004	0	22400	121	2240	1	K	SPAR	36	541
31	2659	061174	122106	0	53312	121	3136	1	INV	K	1	1441
32	4563	061174	122128	0	22400	121	2240	1	M	SPAR	36	541
33	5363	061174	122133	0	10	10	10	-1	VIBR	EVAL	1	0
34	5384	061174	122133	0	7260	121	726	-1	VIBR	U	1	0
35	5624	061174	122536	0	53312	121	3136	1	INV	K	2	1441
36	7528	061174	122543	0	10	10	10	-1	VIBR	EVAL	2	0
37	7529	061174	122543	0	7260	121	726	-1	VIBR	U	2	0
38	7789	061174	123040	0	726	121	726	-1	MFM		0	1
39	7815	061174	123040	0	15	1	15	4	CASE	****	0	1
40	7818	061174	123041	0	726	121	726	-1	MFM		0	2
41	7851	061174	123042	0	15	1	15	4	CASE	****	0	2
42	7852	061174	123042	0	0	0	0	0	LDIR		1	2
43	7852	061174	123209	0	53312	121	3136	1	INV	K	3	1441
44	-9756	061174	123250	0	726	121	726	1	SSOL	U	1	1
45	-9782	061174	123312	0	22400	121	2240	1	KG	SPAR	36	541
46	-10582	061174	123325	0	4	4	4	-1	STAB	EVAL	1	0
47	-10583	061174	123325	0	2904	121	726	-1	STAB	U	1	0
48	10687	061174	123712	0	22400	121	2240	1	KXKG	SPAR	36	541
49	11487	061174	123848	0	53312	121	3136	1	INV	KXKG	3	1441
50	-13391	061174	123900	0	726	121	726	1	SSOL	U	1	2
51	-13417	061174	123918	0	22400	121	2240	1	KG	SPAR	36	541
52	14217	061174	124009	0	53312	121	3136	1	INV	KXKG	1	1441

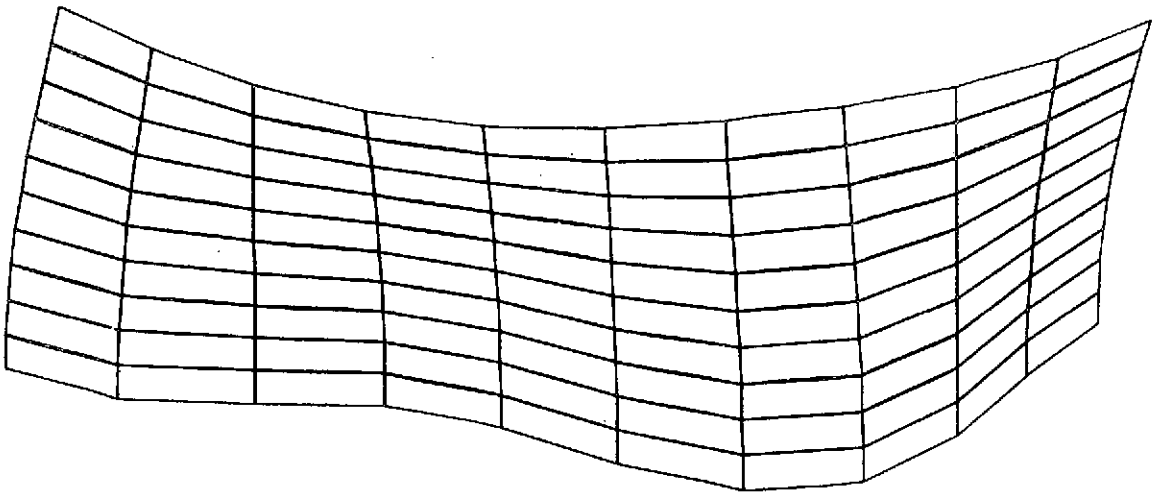
53	-16121	061174	124021	0	4	4	4	-1	STAB	EVAL	1	0	
54	-16122	061174	124021	0	2904	121	726	-1	STAB	U	1	0	
55	16226	061174	124308	0	726	121	726	1	SSOL	U	1	2	
56	-16252	061174	124323	0	22400	121	2240	1	KG	SPAR	36	541	
57	-17052	061174	124327	0	4	4	4	-1	STAB	EVAL	1	0	
58	-17053	061174	124327	0	2904	121	726	-1	STAB	U	1	0	
59	17157	061174	124527	0	22400	121	2240	1	KYKG	SPAR	36	541	
60	17957	061174	124602	0	53312	121	3136	1	INV	KYKG	3	1441	
61	19870	061174	124612	0	726	121	726	1	SSOL	U	1	1	
62	19896	061174	124639	0	22400	121	2240	1	KG	SPAR	36	541	
63	20696	061174	124717	0	53312	121	3136	1	INV	KYKG	1	1441	
64	22600	061174	124731	0	4	4	4	-1	STAB	EVAL	1	0	
65	22601	061174	124731	0	2904	121	726	-1	STAB	U	1	0	

UNDEFORMED STRUCTURE



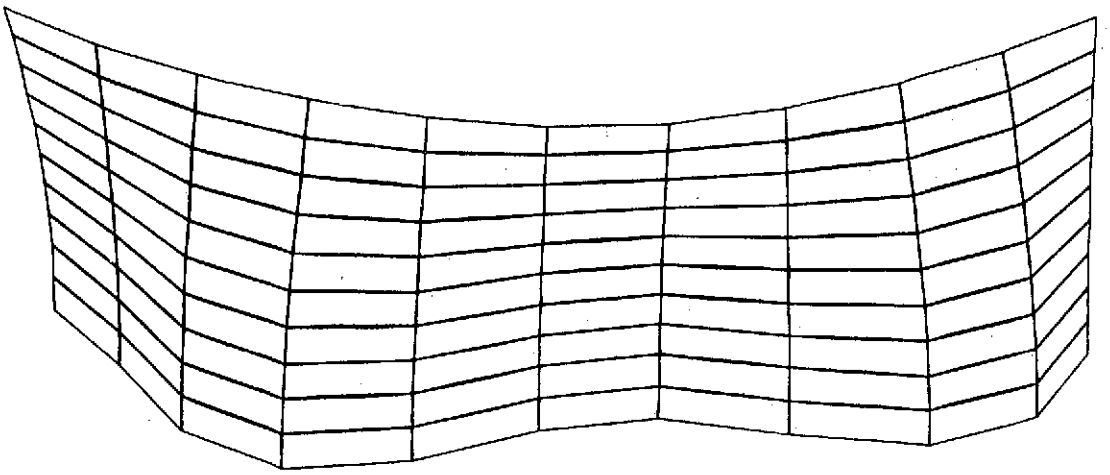
CYLINDRICAL SHELL PROBLEM

SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = .402415X10^{+02}



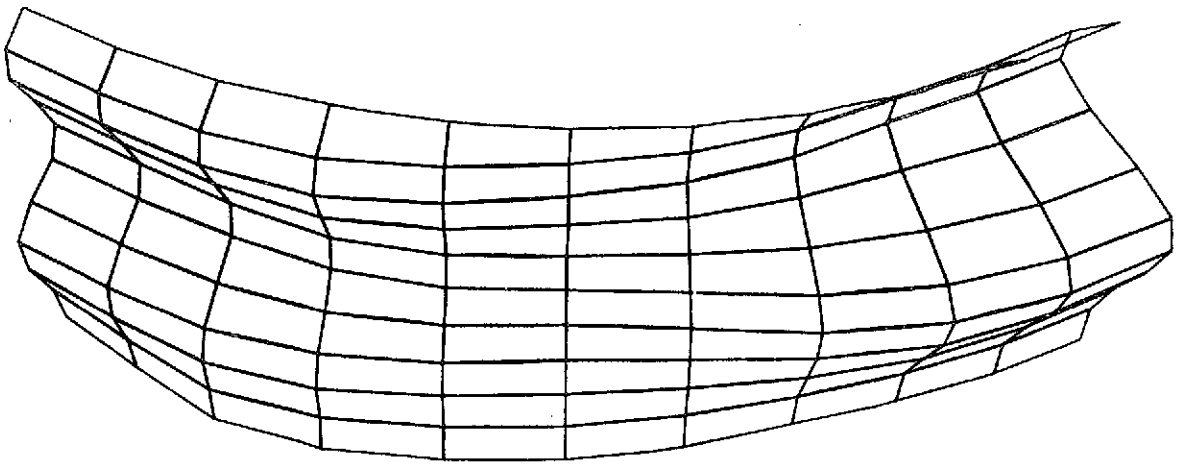
CYLINDRICAL SHELL PROBLEM

SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = .384223X10+02



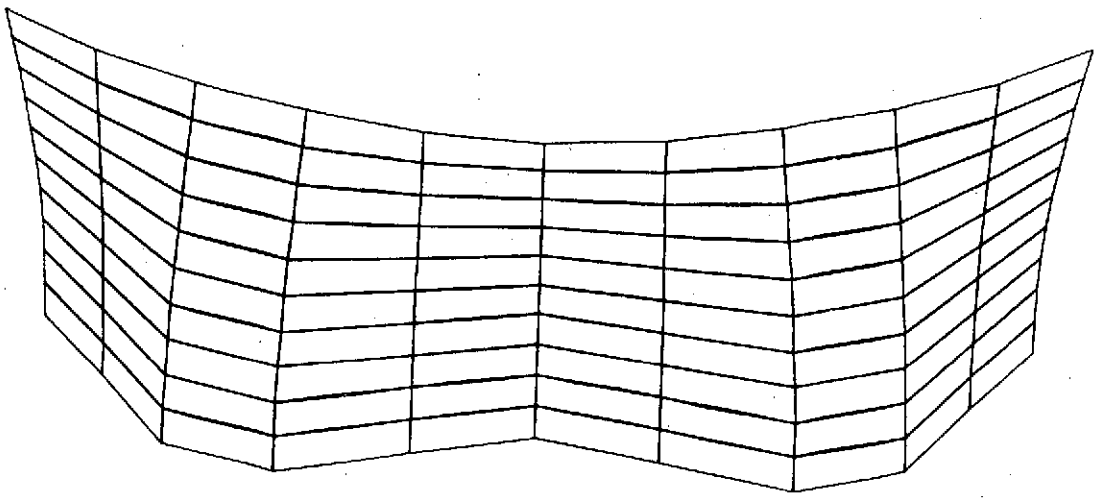
CYLINDRICAL SHELL PROBLEM

SEQ 1 BUCKLING MODE. CRITICAL LOAD = .606797X10^{+04}



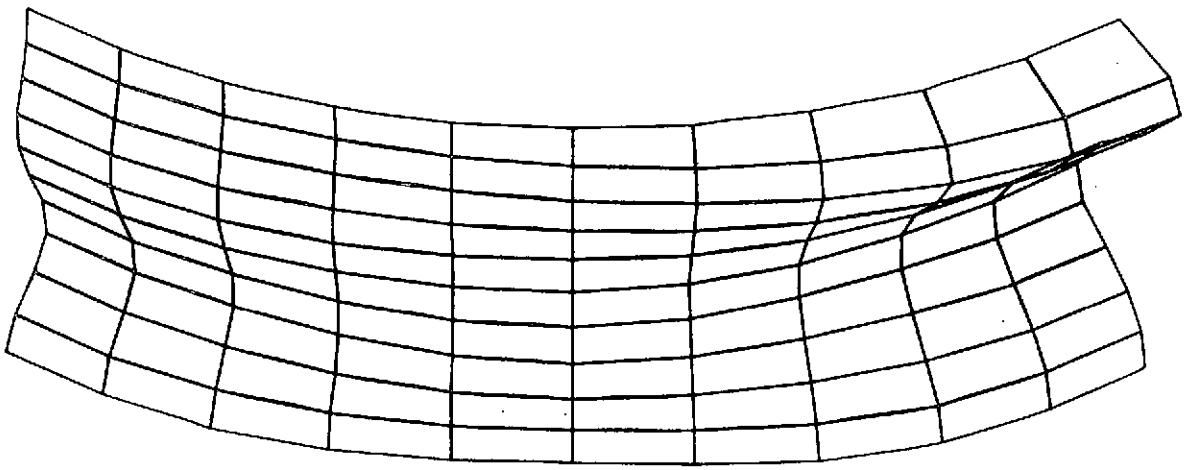
CYLINDRICAL SHELL PROBLEM

SEQ 1 BUCKLING MODE. CRITICAL LOAD = .108406X10+03



CYLINDRICAL SHELL PROBLEM

SEQ 1 BUCKLING MODE. CRITICAL LOAD = .608179X10^{+04}

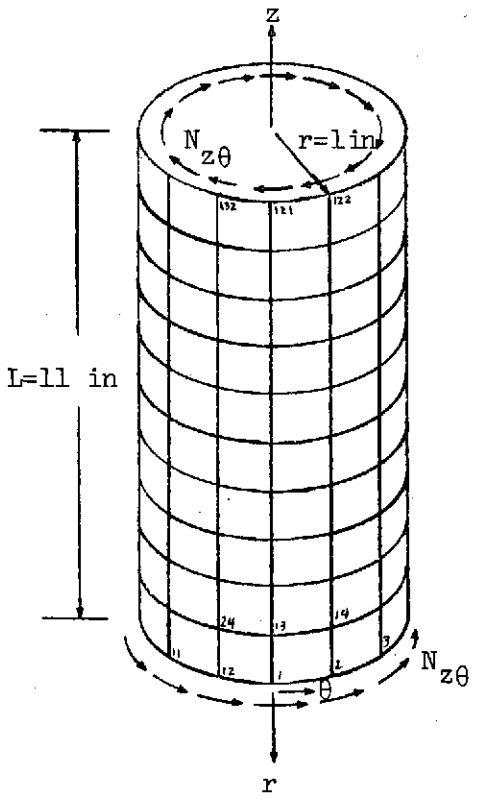


CYLINDRICAL SHELL PROBLEM

0 2
SCALE

7. BUCKLING OF A CYLINDRICAL SHELL DUE TO TORSIONAL LOADING

A simply supported cylindrical shell is subjected to shear load at the ends. The shell is divided into ten elements along the axial direction and twelve elements along the circumferential direction. The geometrical and material properties of the shell are shown on the figure below:



$E = .3 \times 10^8$ lbs/in^2

$\nu = .3$

$t = .1$ in

Results are compared with Ref. 5, p 448, Figure 21.

	$N_{z\theta}$ (Critical)
SPAR	$.3437 \times 10^5$
Ref. 5	$.3423 \times 10^5$

```
@XQT TAB                                    . GENERATE BASIC TABLES DEFINING STRUCTURE
START 132$
TITLE BUCKLING OF A CYLINDRICAL SHELL DUE TO TORSIONAL LOAD.
MATERIAL CONSTANTS$
      1 3.+7 .3 .1 1.-5 1.+4$
JOINT REFERENCE FRAMES$
      NREF=-1$
      1 132 1$
JOINT LOCATIONS$
      FORMAT=2$
       1 1. 0.  0. 1. 330.  0. 12 1 11$
      12 1. 0. 11. 1. 330. 11.$
SHELL SECTION PROPERTIES$
      1 .1$
CONSTRAINT CASE 1$                          CONSTRAIN 1,2 AND 3 DIRECTION MOTION AT
$                                           ONE END OF THE CYLINDER.
$
      ZERO 1 2 3 121 132 1$
CONSTRAINT CASE 2$                          SIMPLY SUPPORTED AT BOTH ENDS OF CYLINDER
$                                           AND SUPPORT AT TWO POINTS AT MIDDLE OF
$                                           THE CYLINDER TO AVOID RIGID BODY MOTION.
      ZERO 1 21  1  12 11 121 132 1$
      ZERO 3  1 61  67 6$
@XQT ELD                                    . READ ELEMENT DEFINITIONS
E43 $
      1 13 14 2  2 12 10$
@XQT TOPO                                   . ANALYZE ELEMENT INTERCONNECTIVITY
$
@XQT E                                      . FORM ELEMENT DATA PACKETS
$
@XQT EKS                                    . INSERT K, S INTO ELEMENT DATA PACKETS
$
@XQT K                                      . FORM SYSTEM K
$
@XQT INV                                    . FACTOR MATRIX IN K SPAR FORMAT
$
@XQT Q                                      . DEFINE APPLIED LOADS
$                                           APPLY UNIFORM SHEAR FORCES AT FREE END TO
$                                           OBTAIN UNIFORM SHEAR STRESS.
      CASE 1$
       NODAL FORCES, MOMENTS$
        1 2 0.523 12 1$
@XQT DSOL                                   . COMPUTE STATIC SOLUTIONS
$
@XQT KG                                     . FORM SYSTEM KG
$
@XQT INV
      RESET CON=2$
@XQT EIG                                    . SOLVE SYSTEM EIGENPROBLEM
$                                           FIND BUCKLING VALUE OF THE SHEAR STRESS.
      RESET PROB=STAB,INIT=R,CON=2,NDYN=10,UPPER=1$
@XQT DCU                                    . EXECUTE DATA COMPLEX UTILITY PROGRAM
      TOC 1$
```

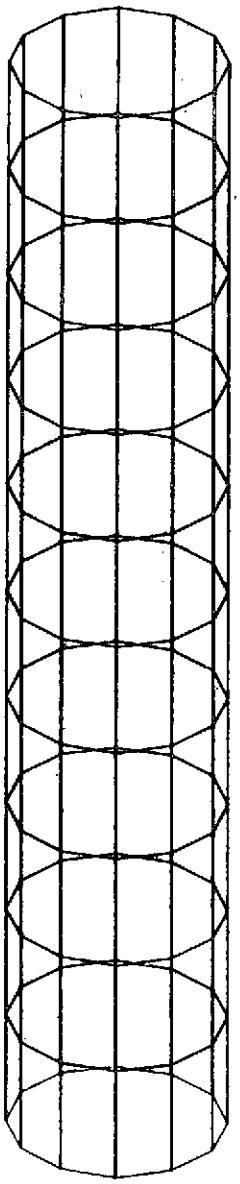

REPRODUCIBILITY OF THE
ORIGINAL PAGE IS POOR

TABLE OF CONTENTS, DAL 1

BUCKLING OF A CYLINDRICAL SHELL DUE TO TORSIONAL LOAD.

SEQ	RR	DATE	TIME	ER	WORDS	ROWS /BLK	BLK SIZE	TY	DATA SET NAME N1	N2	N3	
1	11	061174	073619	0	18	1	18	0	JDFI	BTAB	1	
2	-12	061174	073619	0	132	132	132	0	JREF	BTAB	2	
3	17	061174	073619	0	12	1	12	1	ALTR	BTAB	2	
4	18	061174	073619	0	19	1	19	4	NDAL		0	
5	19	061174	073619	0	10	1	10	1	MATC	BTAB	2	
6	20	061174	073620	0	132	132	132	0	JREF	BTAB	2	
7	25	061174	073621	0	396	132	396	1	JLOC	BTAB	2	
8	40	061174	073622	0	19	1	19	1	SA	BTAB	2	1
9	41	061174	073622	0	132	132	132	0	CON		1	
10	46	061174	073623	0	132	132	132	0	CON		2	
11	51	061174	073623	0	1188	132	1188	1	QJJT	BTAB	2	1
12	94	061174	073631	0	1920	120	896	0	DEF	E43	11	
13	190	061174	073631	0	2	1	2	0	GD	E43	11	
14	191	061174	073631	0	15	1	15	0	GTIT	E43	11	
15	192	061174	073631	0	12	12	12	0	NELZ	BTAB	1	1
16	193	061174	073631	0	5	1	5	0	KE		0	
17	194	061174	073631	0	6	1	6	0	NS		0	
18	195	061174	073631	0	1	1	1	3	ELTS	NAME	0	
19	196	061174	073631	0	1	1	1	0	ELTS	LTYP	0	
20	197	061174	073631	0	1	1	1	0	ELTS	NNOD	0	
21	207	061174	073631	0	1	1	1	0	ELTS	ISCT	0	
22	208	061174	073631	0	1	1	1	0	ELTS	NELS	0	
23	209	061174	073631	0	1	1	1	0	ELTS	LE3	0	
24	210	061174	073640	0	4032	132	1344	0	KMAP		624	3
25	354	061174	073643	0	17472	132	1344	0	AMAP	####	1803	12
26	978	061174	073727	0	36960	120	308	4	E43		11	
27	2298	061174	073648	0	20	20	20	0	DIR	E43	11	
28	2299	061174	073738	0	24640	132	2240	1	K	SPAR	36	68
29	3179	061174	073827	0	75264	132	3136	1	INV	K	1	180
30	5867	061174	074049	0	792	132	792	-1	NFM		0	
31	5896	061174	074049	0	15	1	15	4	CASE	####	0	
32	5897	061174	074049	0	0	0	0	0	LDIR		1	
33	5897	061174	074145	0	792	132	792	1	SSOL	U	1	
34	5926	061174	074249	0	24640	132	2240	1	KG	SPAR	36	68
35	6806	061174	074343	0	75264	132	3136	1	INV	K	2	180
36	9494	061174	074348	0	8	8	8	-1	STAB	EVAL	2	
37	9495	061174	074348	0	6336	132	792	-1	STAB	U	2	

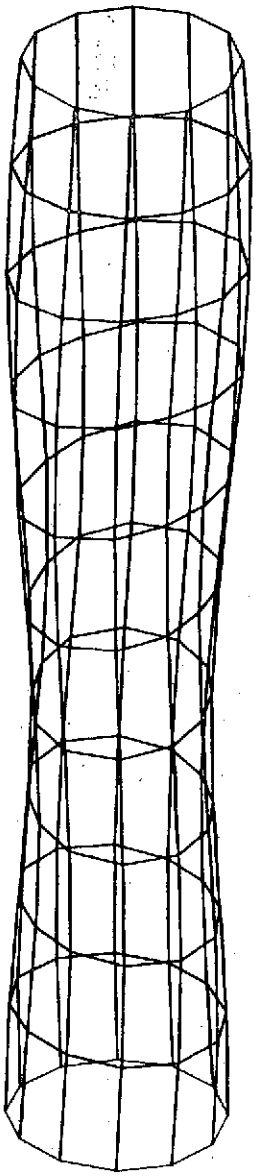
UNDEFORMED STRUCTURE



SHEAR BUCKLING OF A CYLINDRICAL SHELL

0 2
SCALE

SEQ 1 BUCKLING MODE. CRITICAL LOAD = .343730X10+05

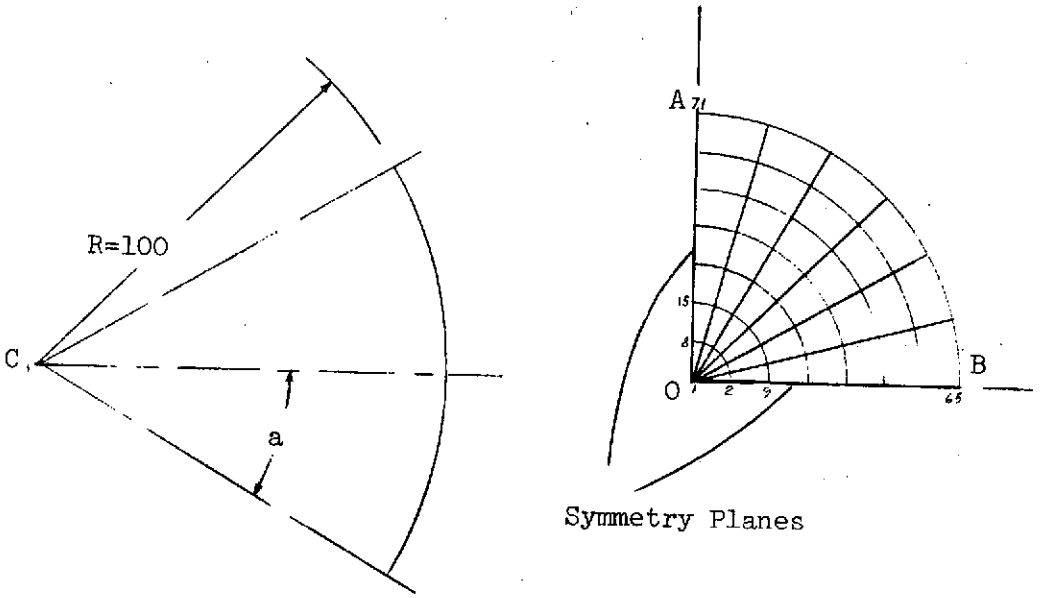


SHEAR BUCKLING OF A CYLINDRICAL SHELL

0
SCALE

8. SPHERICAL SHELL

The 6 by 10 mesh shown on Figure 8-1 was used to solve the problems discussed in Section 8.1 through 8.5



Section	a degrees	t in.	E lb/in^2	ν	Mass Density lb-sec^2/in^4
8.1	60°	5.	10^7	.3	.1
8.2	30°	5.	10^7	.3	.1
8.3	35°	3.	10^7	1/6	
8.4	10°	.396	10^7	1/3	

Fig. 8-1 Spherical Shell

8.1 Vibrational Characteristics of a 60° Sector. With edge AB simply supported, vibrational modes are computed. Results are compared with Reference 6, p 1363, Table 2.

	ω_{01}^2	ω_{02}^2
SPAR	$.9209 \times 10^4$	$.1736 \times 10^5$
Ref. 6	$.9197 \times 10^4$	$.1764 \times 10^5$

8.2 Vibrational Characteristics of a 30° Sector. With edge AB fixed, vibrational modes are computed. Results are compared with Reference 6, p 1363.

	ω_{12}^2
SPAR	$.4743 \times 10^5$
Ref. 6	$.4623 \times 10^5$

8.3 Stresses due to Pressure Loading. With edge AB fixed, unit internal pressure was applied. Results are compared with Reference 4, pp 553-554, Figure 273-274.

	Hoop Stress at Apex	Meridional Moment at AB
SPAR	48.0	35.8
Ref. 4	47.5	36.6

8.4 Buckling of 10° Sector due to Pressure. To obtain a state of uniform compression, unit pressure, P, was applied with displacements normal to conical surface ABC constrained along AB. Edge AB was then fixed, and P (critical) computed. Results are compared with Reference 8, p 450, Figure 6.

	P (Critical)
SPAR	$.236 \times 10^3$
Ref. 7	$.234 \times 10^3$

```
@XQT TAU                                    . GENERATE BASIC TABLES DEFINING STRUCTURE
START 71$
TITLE' SPHERICAL SHELL PROBELMS
TEXT$
!     THE FOLLOWING PROBLEMS IN SPHERICAL SHELL ARE SOLVED IN THIS RUN;
!      1  VIBRATION OF SIMPLY SUPPORTED 60 DEGREE SPHERICAL DOME
!      2  VIBRATION OF FIXED EDGED 30 DEGREE SPHERICAL DOME
!      3  STRESSES IN FIXED EDGED 35 DEGREE SPHERICAL DOME UNDER UNIFORM PRESSURE
!      4  BUCKLING OF FIXED EDGED 10 DEGREE SPHERICAL CAP UNDER UNIFORM PRESSURE
!
MATERIAL CONSTANTS
      1  .1+8  .3      .1   .1-4$
      2  .1+8  .1667   .1   .1-4$
      3  .1+8  .3333   .1   .1-4$
JOINT LOCATIONS$
$                                           JOINT LOCATIONS, 60 DEGREE DOME
      FORMAT=2$
        1   0.0     0.0   0.0
        2  10.453   0.0   0.5478 10.453 90.0  0.5478  7 1
        9  20.791   0.0   2.1852 20.791 90.0  2.1852  7 1
       16  30.902   0.0   4.8944 30.902 90.0  4.8944  7 1
       23  40.674   0.0   8.6455 40.674 90.0  8.6455  7 1
       30  50.0     0.0  13.3975 50.0   90.0 13.3975  7 1
       37  58.779   0.0  19.0983 58.779 90.0 19.0983  7 1
       44  66.913   0.0  25.6855 66.913 90.0 25.6855  7 1
       51  74.315   0.0  33.0869 74.315 90.0 33.0869  7 1
       58  80.902   0.0  41.2215 80.902 90.0 41.2215  7 1
       65  86.603   0.0  50.0    86.603 90.0 50.0     7 1
JOINT REFERENCE FRAME ASSIGNMENTS$
      NREF=-11 1,71$
SHELL SECTION PROPERTIES
      1 5.0$
    NMAT=2
      2 3.0$
    NMAT=3
      3 0.396$
CONSTRAINT CASE 1
$
$                                           SIMPLY SUPPORTED AT EDGES, SYMMETRY AT
                                            THETA=0 AND 90 DEGREE PLANES
      SYMMETRY PLANE=1$
      SYMMETRY PLANE=2$
      ZERO 1,2,3,4,6; 65,71$
CONSTRAINT CASE 2
$
$                                           CONSTRAINED AT EDGES, SYMMETRY AT THETA=0
                                            AND 90 DEGREE PLANES
      SYMMETRY PLANE=1$
      SYMMETRY PLANE=2$
      ZERO 1,2,3,4,5,6; 65,71$
@XQT ELD                                    . READ ELEMENT DEFINITIONS
E43 $
      2 9 10 3  1 9 6$
E33 $
      1 2 3     2 6$
@XQT TOPO                                   . ANALYZE ELEMENT INTERCONNECTIVITY
$
@XQT E                                      . FORM ELEMENT DATA PACKETS
$
@XQT EKS                                    . INSERT K, S INTO ELEMENT DATA PACKETS
$
@XQT M                                      . FORM SYSTEM M
$
```

```
@XQT K                                          . FORM SYSTEM K
$
@XQT INV                                        . FACTOR MATRIX IN K SPAR FORMAT
@XQT EIG                                        . SOLVE SYSTEM EIGENPROBLEM
$                                                 FIND FREQUENCIES AND CORRESPONDING MODE
$                                                 SHAPES FOR FREE VIBRATION OF THE SIMPLY
$                                                 SUPPORTED 60 DEGREE SPHERICAL DOME
      RESET INIT=8$
      PRINT 0 0 0 0 1$
@XQT TAB
JOINT LOCATIONS$
$                                               JOINT LOCATIONS. 30 DEGREE DOME
      FORMAT=2$
      1  0.0     0.0   0.0$
      2  5.2336  0.0   0.1371  5.2336 90.0   0.1371  7 1$
      9 10.4529  0.0   0.5478 10.4529 90.0   0.5478  7 1$
     16 15.6435  0.0   1.2312 15.6435 90.0   1.2312  7 1$
     23 20.7912  0.0   2.1852 20.7912 90.0   2.1852  7 1$
     30 25.8819  0.0   3.4074 25.8819 90.0   3.4074  7 1$
     37 30.9017  0.0   4.8944 30.9017 90.0   4.8944  7 1$
     44 35.8368  0.0   6.6420 35.8368 90.0   6.6420  7 1$
     51 40.6737  0.0   8.6455 40.6737 90.0   8.6455  7 1$
     58 45.3991  0.0  10.8994 45.3991 90.0  10.8994  7 1$
     65 50.0     0.0  13.3975 50.0    90.0  13.3975  7 1$
@XQT E
@XQT EKS
@XQT M
@XQT K
@XQT INV
      RESET CON=2$
@XQT EIG
$                                               FIND VIBRATIONAL MODES OF THE FIXED
$                                               EDGED 30 DEGREE SPHERICAL DOME
      RESET INIT=8,CON=2$
      PRINT 0 0 0 0 1$
@XQT TAB
JOINT LOCATIONS$
$                                               JOINT LOCATIONS. 35 DEGREE DOME
      FORMAT=2$
      1  0.0     0.0   0.0$
      2  5.4944  0.0   0.1679  5.4944 90.0   0.1679  7 1$
      9 10.9682  0.0   0.6709 10.9682 90.0   0.6709  7 1$
     16 16.4012  0.0   1.5071 16.4012 90.0   1.5071  7 1$
     23 21.7730  0.0   2.6734 21.7730 90.0   2.6734  7 1$
     30 27.0635  0.0   4.1655 27.0635 90.0   4.1655  7 1$
     37 32.2531  0.0   5.9778 32.2531 90.0   5.9778  7 1$
     44 37.3224  0.0   8.1035 37.3224 90.0   8.1035  7 1$
     51 42.2525  0.0  10.5347 42.2525 90.0  10.5347  7 1$
     58 47.0249  0.0  13.2624 47.0249 90.0  13.2624  7 1$
     65 51.6218  0.0  16.2763 51.6218 90.0  16.2763  7 1$
@XQT ELD
E43 $
      NSEC=2$
      NMAT=2$
      2 9 10 3  1 9 6$
E33 $
      NSEC=2$
      NMAT=2$
      1 2 3    2 6$
@XQT E
@XQT EKS
```

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```
*XQT K
*XQT INV
      RESET CON=2$
*XQT Q                                  * DEFINE APPLIED LOADS
      CASE 1: UNIFORM PRESSURE LOAD
        NODAL PRESSURE
          1  1.0  71 1$
*XQT DSOL                               * COMPUTE STATIC SOLUTIONS
$                                       FIND DISPLACEMENT OF THE 35 DEGREE DOME
$                                       UNDER UNIFORM PRESSURE
      RESET UP=1,CON=2$
*XQT GSF                                * GENERATE STRESS DATA
$                                       FIND STRESSES OF THE DOME
*XQT PSF                                * PRINT STRESS DATA
$                                       PRINT STRESSES OF THE DOME
*XQT TAB
JOINT LOCATIONS$
$                                       JOINT LOCATIONS, 10 DEGREE DOME
      FORMAT=2$
      1   0.0     0.0   0.0$
      2   1.7452  0.0   0.0152  1.7452  90.0   0.0152   7 1$
      9   3.4900  0.0   0.0609  3.4900  90.0   0.0609   7 1
     16   5.2336  0.0   0.1371  5.2336  90.0   0.1371   7 1$
     23   6.9757  0.0   0.2436  6.9757  90.0   0.2436   7 1$
     30   8.7156  0.0   0.3805  8.7156  90.0   0.3805   7 1$
     37  10.4529  0.0   0.5478 10.4529  90.0   0.5478   7 1$
     44  12.1869  0.0   0.7454 12.1869  90.0   0.7454   7 1$
     51  13.9173  0.0   0.9732 13.9173  90.0   0.9732   7 1$
     58  15.6434  0.0   1.2311 15.6434  90.0   1.2311   7 1$
     65  17.3648  0.0   1.5192 17.3648  90.0   1.5192   7 1$
ALTERNATE REFERENCE FRAME
$                                       DEFINE JOINT REFERENCE FRAMES PARALLEL TO
$                                       SPHERICAL COORDINATE LINES AT EDGES
      2  3   0.0   2 -10.0$
      3  3  15.0   2 -10.0$
      4  3  30.0   2 -10.0$
      5  3  45.0   2 -10.0$
      6  3  60.0   2 -10.0$
      7  3  75.0   2 -10.0$
      8  3  90.0   2 -10.0$
JOINT REFERENCE FRAME ASSIGNMENTS
      NREF=1: 1,64$
      NREF=2: 65$
      NREF=3: 66$
      NREF=4: 67$
      NREF=5: 68$
      NREF=6: 69$
      NREF=7: 70$
      NREF=8: 71$
CONSTRAINT CASE 3
$                                       EDGES ARE SIMPLY SUPPORTED BUT FREE TO
$                                       MOVE IN MERIDIONAL DIRECTION, SYMMETRY AT
$                                       THETA=0 AND 90 DEGREE PLANES
      SYMMETRY PLANE=1$
      SYMMETRY PLANE=2$
      ZERO 1,2,4,6: 65,71$
CONSTRAINT CASE 4
$                                       SAME AS CONSTRAINT CASE 2
      SYMMETRY PLANE=1$
      SYMMETRY PLANE=2$
      ZERO 1,2,3,4,5,6: 65,71$
```

```
@XQT ELD
E43_$
      NSEC=3$
      NMAT=3$
      2 9 10 3  1 9 6$
E33 $
      NSEC=3$
      NMAT=3$
      1 2 3     2 6$
@XQT E
@XQT EKS
@XQT K
@XQT INV
      RESET_CON=3$
@XQT Q
      CASE 1' UNIFORM PRESSURE LOAD
        NODAL PRESSURE
          1  1.0  71 1$
@XQT DSOL
      RESET_CON=3$
@XQT KG                                  , FORM SYSTEM KG
$
@XQT INV
      RESET CON=4$
@XQT EIG
$                                        FIND BUCKLING VALUE OF THE PRESSURE FOR...
$                                        THE 10 DEGREE FIXED EDGED SPHERICAL CAP
      RESET PROB=STAB,CON=4,INIT=4,NDYN=10,UPPER=1$
      PRINT 0 0 0 0 1$
@XQT DCU
      TOC 1$
```

TABLE OF CONTENTS, DAL 1

SPHERICAL SHELL PROBELMS

SEQ	RR	DATE	TIME	ER	WORDS	ROWS /BLK	BLK SIZE	TY	DATA SET NAME N1	N2	N3	N4
1	11	051574	171123	0	18	1	18	0	JDF1	BTAB	1	8
2	-12	051574	171123	0	71	71	71	0	JREF	BTAB	2	6
3	-15	051574	171123	0	12	1	12	1	ALTR	BTAB	2	4
4	16	051574	171123	0	19	1	19	4	NDAL		0	0
5	17	051574	171124	0	80	4	80	3	TEXT	BTAB	2	1
6	20	051574	171124	0	30	3	30	1	MATC	BTAB	2	2
7	-22	051574	171124	0	213	71	213	1	JLOC	BTAB	2	5
8	-30	051574	171124	0	71	71	71	0	JREF	BTAB	2	6
9	33	051574	171125	0	57	3	57	1	SA	BTAB	2	13
10	36	051574	171126	0	71	71	71	0	CON		1	0
11	39	051574	171126	0	71	71	71	0	CON		2	0
12	-42	051574	171127	0	639	71	639	1	QJJT	BTAB	2	19
13	-65	051574	171204	0	864	54	896	0	DEF	E43	11	4
14	-97	051574	171204	0	2	1	2	0	GD	E43	11	4
15	-98	051574	171204	0	15	1	15	0	GTIT	E43	11	4
16	-99	051574	171204	0	96	6	896	0	DEF	E33	8	3
17	-131	051574	171204	0	2	1	2	0	GD	E33	8	3
18	-132	051574	171204	0	15	1	15	0	GTIT	E33	8	3
19	-133	051574	171204	0	12	12	12	0	NELZ	BTAB	1	11
20	-134	051574	171204	0	10	2	10	0	KE		0	0
21	-144	051574	171205	0	12	2	12	0	NS		0	0
22	-145	051574	171205	0	2	2	2	3	ELTS	NAME	0	0
23	-146	051574	171205	0	2	2	2	0	ELTS	LTYP	0	0
24	-147	051574	171206	0	2	2	2	0	ELTS	NNOD	0	0
25	-148	051574	171206	0	2	2	2	0	ELTS	ISCT	0	0
26	-149	051574	171206	0	2	2	2	0	ELTS	NELS	0	0
27	-150	051574	171207	0	2	2	2	0	ELTS	LE3	0	0
28	151	051574	171225	0	2688	71	1344	0	KMAP		309	21
29	247	051574	171227	0	4032	71	1344	0	AMAP	****	594	45
30	-391	051574	171307	0	1344	6	224	4	E33		8	3
31	-439	051574	171251	0	20	20	20	0	DIR	E33	8	3
32	-440	051574	171311	0	16632	54	308	4	E43		11	4
33	-1034	051574	171252	0	20	20	20	0	DIR	E43	11	4
34	-1035	051574	171325	0	13440	71	2240	1	M	SPAR	36	309
35	-1515	051574	171335	0	13440	71	2240	1	K	SPAR	36	309
36	1995	051574	171358	0	21952	71	3136	1	INV	K	1	594
37	2779	051574	171404	0	8	8	8	-1	VIBR	EVAL	1	0
38	2780	051574	171404	0	3408	71	426	-1	VIBR	U	1	0
39	-2908	051574	171700	0	213	71	213	1	JLOC	BTAB	2	5
40	-2916	051574	171700	0	639	71	639	1	QJJT	BTAB	2	19
41	-2948	051574	171722	0	1344	6	224	4	E33		8	3
42	-2996	051574	171705	0	20	20	20	0	DIR	E33	8	3
43	-2997	051574	171727	0	16632	54	308	4	E43		11	4
44	-3591	051574	171707	0	20	20	20	0	DIR	E43	11	4
45	3592	051574	171746	0	13440	71	2240	1	M	SPAR	36	309
46	-4072	051574	171753	0	13440	71	2240	1	K	SPAR	36	309
47	-4552	051574	171858	0	21952	71	3136	1	INV	K	2	594
48	5336	051574	171902	0	8	8	8	-1	VIBR	EVAL	2	0
49	5337	051574	171903	0	3408	71	426	-1	VIBR	U	2	0
50	-5465	051574	172107	0	213	71	213	1	JLOC	BTAB	2	5
51	-5473	051574	172109	0	639	71	639	1	QJJT	BTAB	2	19
52	-5496	051574	172127	0	864	54	896	0	DEF	E43	11	4

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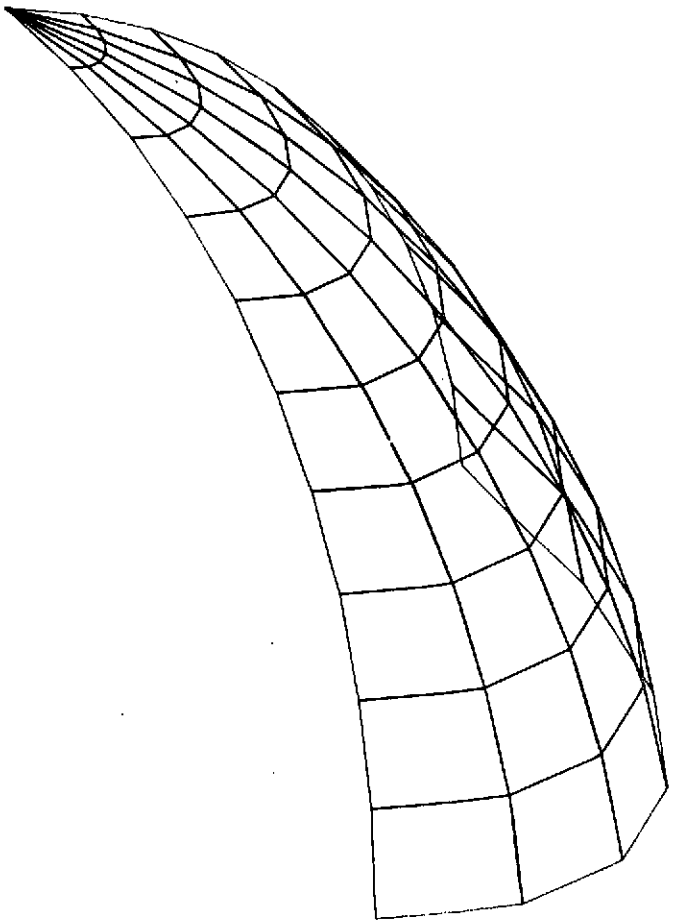
53	-5528	051574	172128	0	2	1	2	0	GD	E43	11	4
54	-5529	051574	172128	0	15	1	15	0	GTIT	E43	11	4
55	-5530	051574	172129	0	96	6	896	0	DEF	E33	8	3
56	-5562	051574	172130	0	2	1	2	0	GD	E33	8	3
57	-5563	051574	172130	0	15	1	15	0	GTIT	E33	8	3
58	-5564	051574	172130	0	12	12	12	0	NELZ	BTAB	1	11
59	-5565	051574	172131	0	10	2	10	0	KE		0	0
60	-5566	051574	172131	0	12	2	12	0	NS		0	0
61	-5576	051574	172132	0	2	2	2	3	ELTS	NAME	0	0
62	-5577	051574	172133	0	2	2	2	0	ELTS	LTYP	0	0
63	-5578	051574	172133	0	2	2	2	0	ELTS	NNOD	0	0
64	-5579	051574	172133	0	2	2	2	0	ELTS	ISCT	0	0
65	-5580	051574	172134	0	2	2	2	0	ELTS	NELS	0	0
66	-5581	051574	172134	0	2	2	2	0	ELTS	LE3	0	0
67	-5582	051574	172158	0	1344	6	224	4	E33		8	3
68	-5630	051574	172148	0	20	20	20	0	DIR	E33	8	3
69	-5631	051574	172204	0	16632	54	308	4	E43		11	4
70	-6225	051574	172151	0	20	20	20	0	DIR	E43	11	4
71	-6226	051574	172212	0	426	71	426	-1	NFM		0	1
72	-6242	051574	172212	0	15	1	15	4	CASE	####	0	1
73	-6243	051574	172212	0	0	0	0	0	LDIR		1	1
74	-6243	051574	172220	0	13440	71	2240	1	K	SPAR	36	309
75	6723	051574	172231	0	21952	71	3136	1	INV	K	2	594
76	-7507	051574	172248	0	426	71	426	1	SSOL	U	0	1
77	-7523	051574	172309	0	186	6	5600	-1	STRS	E33	0	1
78	-7723	051574	172310	0	2646	54	5600	-1	STRS	E43	0	1
79	7923	051574	172325	0	213	71	213	1	JLOC	BTAB	2	5
80	7931	051574	172326	0	96	8	96	1	ALTR	BTAB	2	4
81	7944	051574	172328	0	71	71	71	0	JREF	BTAB	2	6
82	7947	051574	172330	0	71	71	71	0	CON		3	0
83	7950	051574	172331	0	71	71	71	0	CON		4	0
84	7953	051574	172333	0	639	71	639	1	QJJT	BTAB	2	19
85	7976	051574	172340	0	864	54	896	0	DEF	E43	11	4
86	8008	051574	172340	0	2	1	2	0	GD	E43	11	4
87	8009	051574	172341	0	15	1	15	0	GTIT	E43	11	4
88	8010	051574	172342	0	96	6	896	0	DEF	E33	8	3
89	8042	051574	172342	0	2	1	2	0	GD	E33	8	3
90	8043	051574	172343	0	15	1	15	0	GTIT	E33	8	3
91	8044	051574	172344	0	12	12	12	0	NELZ	BTAB	1	11
92	8045	051574	172345	0	10	2	10	0	KE		0	0
93	8046	051574	172345	0	12	2	12	0	NS		0	0
94	8047	051574	172346	0	2	2	2	3	ELTS	NAME	0	0
95	8048	051574	172347	0	2	2	2	0	ELTS	LTYP	0	0
96	8049	051574	172348	0	2	2	2	0	ELTS	NNOD	0	0
97	8050	051574	172348	0	2	2	2	0	ELTS	ISCT	0	0
98	8051	051574	172349	0	2	2	2	0	ELTS	NELS	0	0
99	8052	051574	172350	0	2	2	2	0	ELTS	LE3	0	0
100	8053	051574	172403	0	1344	6	224	4	E33		8	3
101	8110	051574	172357	0	20	20	20	0	DIR	E33	8	3
102	8111	051574	172409	0	16632	54	308	4	E43		11	4
103	8705	051574	172400	0	20	20	20	0	DIR	E43	11	4
104	8706	051574	172433	0	13440	71	2240	1	K	SPAR	36	309
105	9186	051574	172449	0	21952	71	3136	1	INV	K	3	594
106	9970	051574	172510	0	426	71	426	-1	NFM		0	1
107	9986	051574	172510	0	15	1	15	4	CASE	####	0	1
108	9987	051574	172511	0	0	0	0	0	LDIR		1	1
109	9987	051574	172529	0	426	71	426	1	SSOL	U	0	1
110	10003	051574	172550	0	186	6	5600	-1	STRS	E33	0	1

111	10203	051574	172551	0	2646	54	5600	-1	STRS	E43	0	1
112	10403	051574	172615	0	13440	71	2240	1	KG	SPAR	36	309
113	10883	051574	172628	0	21952	71	3136	1	INV	K	4	594
114	11667	051574	172640	0	4	4	4	-1	STAB	EVAL	4	0
115	11668	051574	172641	0	1704	71	426	-1	STAB	U	4	0

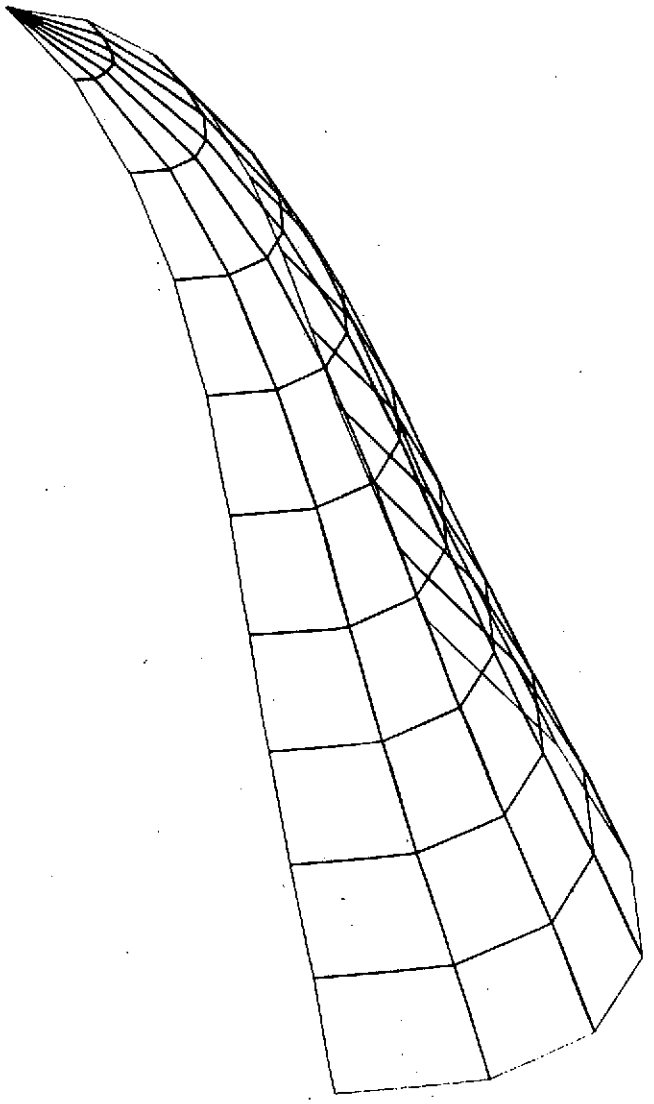
UNDEFORMED STRUCTURE



60-DEGREE
SPHERICAL SHELL

0 SCALE 18

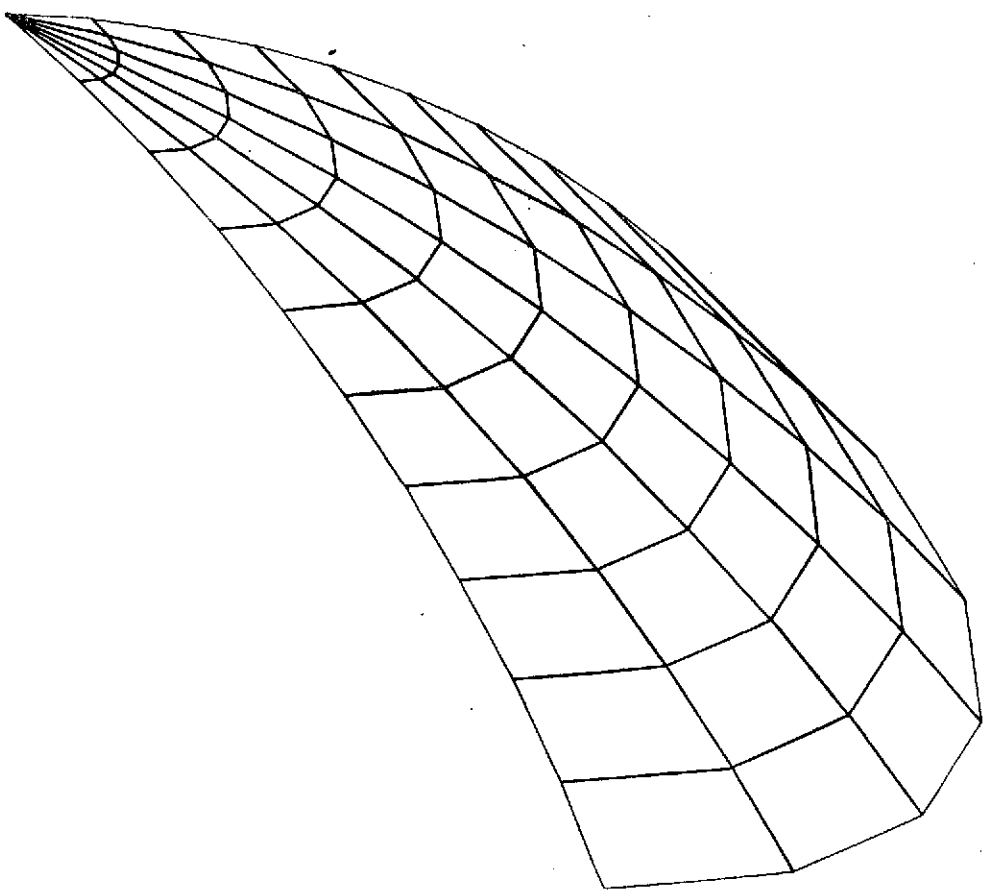
SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = .152732X10+02



60-DEGREE
SPHERICAL SHELL

0
SCALE

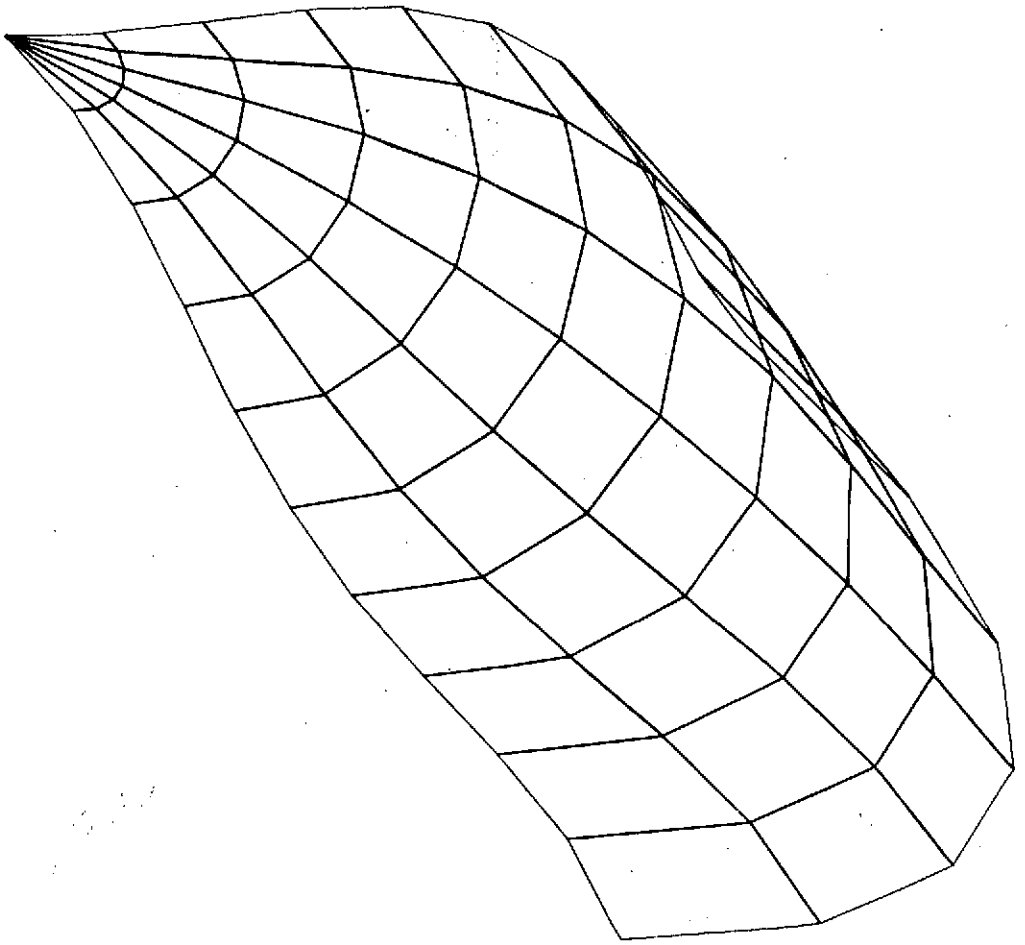
UNDEFORMED STRUCTURE



30-DEGREE
SPHERICAL SHELL

0 8
SCALE

SEQ 2 VIBRATIONAL MODE. FREQ (HZ) = .346612X10+02

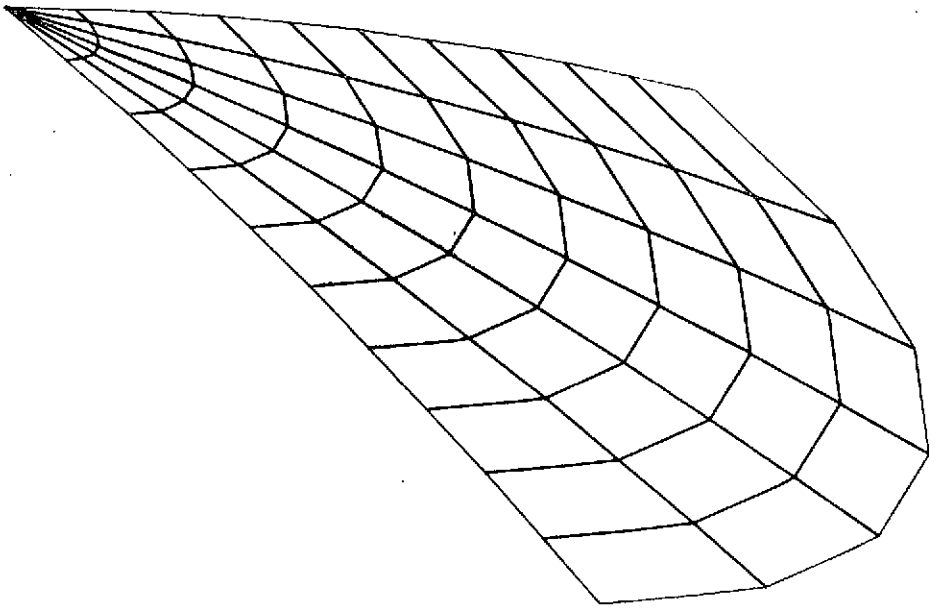


30-DEGREE

SPHERICAL SHELL

0 SCALE 8

UNDEFORMED STRUCTURE

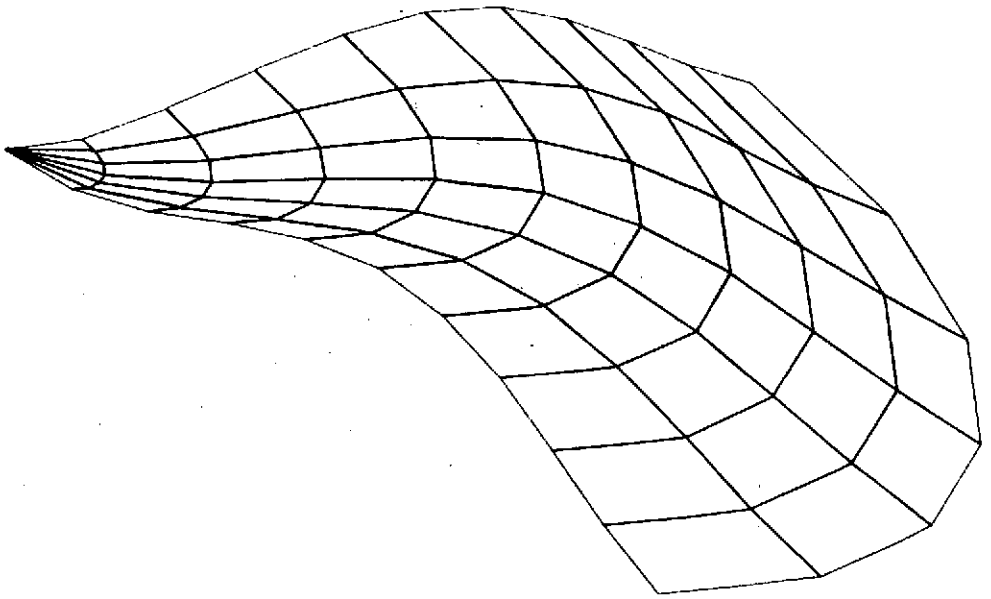


10-DEGREE

SPHERICAL SHELL

0 SCALE

SEQ 1 BUCKLING MODE. CRITICAL LOAD = .236468X10^{+03}



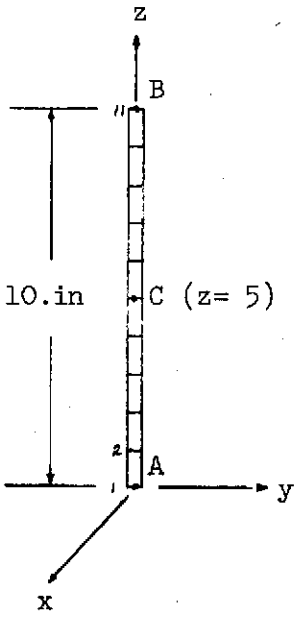
10-DEGREE

SPHERICAL SHELL

0
SCALE

9. BEAM

The 10 element mesh shown on Figure 9-1 was used to solve the problems discussed in Section 9.1 through 9.8.



$E = 10^7$ lb/in^2

$G = .5 \times 10^7$ lb/in^2

Mass density = .1 lb-sec^2/in^4

$\alpha = .1 \times 10^{-4}$ in/degree

$I_x = .36 \times 10^{-2}$ in^4

$I_y = .4 \times 10^{-3}$ in^4

$I_z = .4 \times 10^{-2}$ in^4

$J = 0.16 \times 10^{-2}$ in^4

$A = .12$ in^2

Fig. 9-1 Beam

9.1 Vibration of a Cantilever. With point A fixed and the beam constrained to move in the yz plane, vibrational modes were computed. Results are compared with Reference 1, p 339.

	SPAR	Ref. 2
ω_1^2	$.41213 \times 10^3$	$.41199 \times 10^3$
ω_2^2	$.16185 \times 10^5$	$.16182 \times 10^5$
ω_3^2	$.12695 \times 10^6$	$.12690 \times 10^6$
ω_4^2	$.48817 \times 10^6$	$.48732 \times 10^6$
ω_5^2	$.13382 \times 10^7$	$.13314 \times 10^7$

9.2 Column Buckling. With point A fixed, the lowest critical value of compressive load at B was computed.

SPAR	$.98695 \times 10^2$
$\frac{EI_y \pi^2}{4\ell^2}$	$.98695 \times 10^2$

9.3 Beam-Column Effects. A compressive pre-load of 1000 lbs was applied. With simple support at A and B, lateral displacement under a 10 lb lateral load at C was computed.

	Lateral Deflection at C
SPAR	$.80231 \times 10^{-2}$
Ref. 8	$.80233 \times 10^{-2}$

9.4 Effects of Pre-Tension on Vibration. A tensile pre-load of 1000 lbs was applied. With simple support at A and B, vibrational modes were computed. Results are compared with Reference 2, pp 374-376.

	SPAR	Ref. 2
ω_1^2	$.11472 \times 10^5$	$.11472 \times 10^5$
ω_2^2	$.84861 \times 10^6$	$.84850 \times 10^6$
ω_3^2	$.33731 \times 10^6$	$.33703 \times 10^6$
ω_4^2	$.96559 \times 10^6$	$.96282 \times 10^6$
ω_5^2	$.22511 \times 10^7$	$.22350 \times 10^7$

9.5 Buckling of a Cantilever due to Lateral Load. With A fixed, the critical value of an x-direction point force at B was computed. Results are compared with Reference 3, pp 259-260.

	Critical Force
SPAR	$.22760 \times 10^3$
Ref. 3	$.22674 \times 10^3$

9.6 Buckling due to Pure Bending. Critical values of equilibrating point moments(about the y axis) applied at A and B are computed. Results are compared to Reference 2, pp 253-257.

	Critical Moment
SPAR	$.17845 \times 10^4$
Ref. 3	$.17750 \times 10^4$

9.7 Buckling due to Thermal Load. With A and B simply supported, the critical temperature was computed.

	T (Critical)
SPAR	$.32899 \times 10^2$
$\frac{EI\pi^2}{L\ AE\alpha}$	$.32899 \times 10^2$

9.8 Effects of Thermal Pre-Stress on Vibration. A 10^{o} temperature load was applied to the beam, with A and B simply supported. Vibrational modes were computed. Results are compared with Reference 2, pp 374-375

	SPAR	Ref. 2
ω_1^2	$.22600 \times 10^4$	$.22600 \times 10^4$
ω_2^2	$.48015 \times 10^5$	$.48004 \times 10^5$
ω_3^2	$.25440 \times 10^6$	$.25412 \times 10^6$
ω_4^2	$.81819 \times 10^6$	$.81543 \times 10^6$
ω_5^2	$.20208 \times 10^7$	$.20047 \times 10^6$

```
@XQT TAB                                . GENERATE BASIC TABLES DEFINING STRUCTURE
START 11$
TITLE!  BEAM PROBLEMS
TEXT$
!      THE FOLLOWING BEAM PROBLEMS ARE SOLVED IN THIS RUN:
!       1  VIBRATION OF A CANTILEVERED BEAM
!       2  BUCKLING OF A COLUMN
!       3  LATERAL DEFLECTION OF A COMPRESSED BEAM
!       4  VIBRATION OF A PRESTRESSED BEAM
!       5  LATERAL BUCKLING OF A CANTILEVERED BEAM
!       6  LATERAL BUCKLING OF A BEAM IN PURE BENDING
!       7  BUCKLING OF A BEAM DUE TO TEMPERATURE
!       8  VIBRATION OF A HEATED BEAM
!
MATERIAL CONSTANTS
    1  .1+8  .0  .1  .1-4$
JOINT LOCATIONS
    1  .0  .0  .0  .0  .0  10.0  11 1$
BEAM ORIENTATION SPECIFICATIONS$
    1  1  2  1  0.$
E21 SECTION PROPERTIES$
    GIVN  1  .0004  .0  .0036  .0  .12  .0016$
CONSTRAINT CASE 1$
$                                       THE BEAM IS CONSTRAINED AT ONE END
    FIXED PLANE = 3$
CONSTRAINT CASE 2$
$                                       SIMPLY SUPPORTED AT BOTH ENDS
    ZERO 1,2,3,6!   1,11,10$
@XQT ELD                                . READ ELEMENT DEFINITIONS
E21 $
    1 2  1 10$
@XQT TOPO                               . ANALYZE ELEMENT INTERCONNECTIVITY
$
@XQT E                                  . FORM ELEMENT DATA PACKETS
$
@XQT EKS                                . INSERT K, S INTO ELEMENT DATA PACKETS
$
@XQT M                                  . FORM SYSTEM M
$                                       EXCLUDE ROTATORY INERTIA EFFECTS
    RESET IBEAM=1$
@XQT K                                  . FORM SYSTEM K
$
@XQT INV                                . FACTOR MATRIX IN K SPAR FORMAT
$
@XQT EIG                                . SOLVE SYSTEM EIGENPROBLEM
$                                       FIND VIBRATIONAL MODES OF THE CANTILEVER
$                                       BEAM.
    RESET INIT=10,REPR=1$
@XQT Q                                  . DEFINE APPLIED LOADS
    CASE 1! AXIAL COMPRESSIVE FORCE APPLY AT FREE END.
      NODAL FORCES, MOMENTS
        11 3 -1.0$
    CASE 2! SHEAR FORCE IN 1-DIRECTION APPLY AT FREE END.
      NODAL FORCES, MOMENTS
        11 1 1.0$
    CASE 3! BENDING MOMENT IN 2-DIRECTION APPLY AT BOTH ENDS.
      NODAL FORCES, MOMENTS
         1 5 1.0!  11 5 -1.0$
    CASE 4! CONCENTRATED FORCE AT MID-SPAN
      NODAL FORCES, MOMENTS
         6 1 10.0$
```

REPRODUCIBILITY OF THE
ORIGINAL PAGE IS POOR

```
      CASE 5: TEMPERATURE LOAD AT ALL JOINTS.
        NODAL TEMPERATURES
            1  1.0  11  1$
  @XQT DSOL                                 . SOLVE LOAD CASE 1
       RESET L1=1,L2=1$
  @XQT KG                                   . FORM SYSTEM KG, AXIAL COMPRESSION
  $
  @XQT EIG
  $                                           FIND BUCKLING LOAD FOR CANTILEVERED
  $                                           COLUMN
       RESET INIT=10,PROB=STAB$
  @XQT LCM                                  . FORM LINEAR COMBINATION OF MATRICES
  $                                           FORM STIFFNESS MATRIX FOR AXIALY PRE-
  $                                           COMPRESSED BEAM, COMPRESSIVE FORCE=1000.
       RESET R=KXKG,Q1=K,Q2=KG,C2=1000.0$
  @XQT INV
       RESET K=KXKG,CON=2$
  @XQT DSOL                                 . SOLVE LOAD CASE 4
  $                                           FIND DEFLECTION OF THE PRESTRESSED BEAM
  $                                           UNDER LOAD CASE 4.
       RESET K=KXKG,CON=2,L1=4,L2=4$
  @XQT LCM
  $                                           FORM STIFFNESS MATRIX FOR AXIALY PRE-
  $                                           STRESSED BEAM, TENSILE FORCE=1000.
       RESET R=KYKG,Q1=K,Q2=KG,C2=-1000.0$
  @XQT INV
       RESET K=KYKG,CON=2$
  @XQT EIG
  $                                           FIND FREQUENCY FOR FREE VIBRATION OF THE
  $                                           PRE-STRESSED BEAM.
       RESET K=KYKG,CON=2,INIT=10$
  @XQT DSOL                                 . SOLVE LOAD CASE 2
       RESET L1=2,L2=2$
  @XQT KG                                   . FORM SYSTEM KG, LOAD CASE 2
       RESET QSEQ=2$
  @XQT EIG
  $                                           FIND LATERAL BUCKLING VALUE OF LOAD CASE
  $                                           2 FOR THE CANTILEVER BEAM.
       RESET INIT=6,PROB=STAB$
  @XQT INV
       RESET CON=2$
  @XQT DSOL
       RESET CON=2,L1=3,L2=3$
  @XQT KG                                   . FORM SYSTEM KG, PURE BENDING
       RESET QSEQ=3$
  @XQT EIG
  $                                           FIND BUCKLING LOAD OF A SIMPLY SUPPORTED
  $                                           BEAM IN PURE BENDING.
       RESET INIT=6,PROB=STAB,CON=2$
  @XQT DSOL
       RESET L1=5,L2=5,CON=2$
  @XQT KG                                   . FORM SYSTEM KG, TEMPERATURE LOAD
       RESET QSEQ=5$
  @XQT EIG
  $                                           FIND BUCKLING TEMPERATURE LOAD FOR A
  $                                           SIMPLY SUPPORTED BEAM.
       RESET CON=2,INIT=6,PROB=STAB$
  @XQT LCM
       RESET R=K5KG,Q1=K,Q2=KG,C2=10.0$
  @XQT INV
       RESET K=K5KG,CON=2$
```

```
*XQT EIG
$                                       FIND FREQUENCIES FOR FREE VIBRATION OF
$                                       THE HEATED BEAM
      RESET K=K5KG,CON=2,INIT=10$
      PRINT 0 0 0 0 1$
*XQT DCU                                . EXECUTE DATA COMPLEX UTILITY PROGRAM
      TOC 1$
```

TABLE OF CONTENTS, DAL 1

BEAM PROBLEMS

SEQ	RR	DATE	TIME	ER	WORDS	ROWS /BLK	BLK SIZE	TY	DATA SET NAME N1	N2	N3	N4
1	11	061274	160919	0	18	1	18	0	JDF1	BTAB	1	8
2	12	061274	160919	0	11	11	11	0	JREF	BTAB	2	6
3	13	061274	160919	0	12	1	12	1	ALTR	BTAB	2	4
4	14	061274	160919	0	19	1	19	4	NDAL		0	0
5	15	061274	160919	0	200	10	200	3	TEXT	BTAB	2	1
6	23	061274	160920	0	10	1	10	1	MATC	BTAB	2	2
7	24	061274	160921	0	33	11	33	1	JLOC	BTAB	2	5
8	26	061274	160921	0	5	1	5	1	MREF	BTAB	2	7
9	27	061274	160921	0	31	1	31	1	BA	BTAB	2	9
10	29	061274	160922	0	11	11	11	0	CON		1	0
11	30	061274	160922	0	11	11	11	0	CON		2	0
12	31	061274	160922	0	99	11	99	1	QJJT	BTAB	2	19
13	35	061274	160930	0	160	10	896	0	DEF	E21	1	2
14	67	061274	160930	0	2	1	2	0	GD	E21	1	2
15	68	061274	160930	0	15	1	15	0	GTIT	E21	1	2
16	69	061274	160930	0	12	12	12	0	NELZ	BTAB	1	11
17	70	061274	160930	0	5	1	5	0	KE		0	0
18	71	061274	160930	0	6	1	6	0	NS		0	0
19	72	061274	160930	0	1	1	1	3	ELTS	NAME	0	0
20	73	061274	160930	0	1	1	1	0	ELTS	LTYP	0	0
21	83	061274	160930	0	1	1	1	0	ELTS	NNOD	0	0
22	84	061274	160930	0	1	1	1	0	ELTS	ISCT	0	0
23	85	061274	160930	0	1	1	1	0	ELTS	NELS	0	0
24	86	061274	160931	0	1	1	1	0	ELTS	LE3	0	0
25	87	061274	160937	0	1344	11	1344	0	KMAP		21	3
26	135	061274	160939	0	1344	11	1344	0	AMAP	****	21	3
27	183	061274	160949	0	1400	10	140	4	E21		1	2
28	233	061274	160946	0	20	20	20	0	DIR	E21	1	2
29	234	061274	160953	0	2240	11	2240	1	M	SPAR	36	21
30	314	061274	160956	0	2240	11	2240	1	K	SPAR	36	21
31	394	061274	161010	0	3136	11	3136	1	INV	K	1	21
32	506	061274	161024	0	10	10	10	-1	VIBR	EVAL	1	0
33	507	061274	161024	0	660	11	66	-1	VIBR	U	1	0
34	537	061274	161101	0	66	11	66	-1	NFM		0	1
35	540	061274	161101	0	15	1	15	4	CASE	****	0	1
36	541	061274	161101	0	66	11	66	-1	NFM		0	2
37	544	061274	161101	0	15	1	15	4	CASE	****	0	2
38	545	061274	161101	0	66	11	66	-1	NFM		0	3
39	548	061274	161101	0	15	1	15	4	CASE	****	0	3
40	549	061274	161101	0	66	11	66	-1	NFM		0	4
41	561	061274	161102	0	15	1	15	4	CASE	****	0	4
42	562	061274	161102	0	11	11	11	-1	NTMP		0	5
43	563	061274	161102	0	66	11	66	-1	NFM		0	5
44	566	061274	161103	0	15	1	15	4	CASE	****	0	5
45	567	061274	161103	0	0	0	0	0	LDIR		1	5
46	567	061274	161112	0	66	11	66	1	SSOL	U	1	1
47	-570	061274	161116	0	2240	11	2240	1	KG	SPAR	36	21
48	-650	061274	161121	0	10	10	10	-1	STAB	EVAL	1	0
49	-651	061274	161121	0	660	11	66	-1	STAB	U	1	0
50	681	061274	161150	0	2240	11	2240	1	KXKG	SPAR	36	21
51	761	061274	161157	0	3136	11	3136	1	INV	KXKG	2	21
52	873	061274	161203	0	66	11	66	1	SSOL	U	1	4

53	876	061274	161208	0	2240	11	2240	1	KYKG	SPAR	36	21
54	956	061274	161212	0	3136	11	3136	1	INV	KYKG	2	21
55	-1068	061274	161227	0	10	10	10	-1	VIBR	EVAL	2	0
56	-1069	061274	161227	0	660	11	66	-1	VIBR	U	2	0
57	1099	061274	161302	0	66	11	66	1	SSOL	U	1	2
58	-1102	061274	161307	0	2240	11	2240	1	KG	SPAR	36	21
59	1182	061274	161314	0	6	6	6	-1	STAB	EVAL	1	0
60	1183	061274	161314	0	396	11	66	-1	STAB	U	1	0
61	1210	061274	161336	0	3136	11	3136	1	INV	K	2	21
62	1322	061274	161342	0	66	11	66	1	SSOL	U	1	3
63	-1325	061274	161348	0	2240	11	2240	1	KG	SPAR	36	21
64	-1405	061274	161441	0	6	6	6	-1	STAB	EVAL	2	0
65	-1406	061274	161441	0	396	11	66	-1	STAB	U	2	0
66	1424	061274	161519	0	66	11	66	1	SSOL	U	1	5
67	1427	061274	161522	0	2240	11	2240	1	KG	SPAR	36	21
68	1507	061274	161528	0	6	6	6	-1	STAB	EVAL	2	0
69	1508	061274	161528	0	396	11	66	-1	STAB	U	2	0
70	1526	061274	161554	0	2240	11	2240	1	K5KG	SPAR	36	21
71	1606	061274	161558	0	3136	11	3136	1	INV	K5KG	2	21
72	1718	061274	161605	0	10	10	10	-1	VIBR	EVAL	2	0
73	1719	061274	161606	0	660	11	66	-1	VIBR	U	2	0

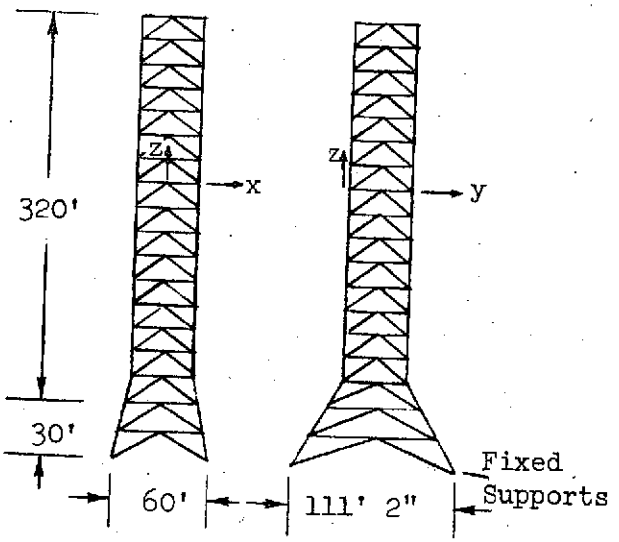
10. LUT (SATURN 5 LAUNCHER UMBILICAL TOWER)

Inertia prestress vibrational modes were calculated for the LUT. This problem illustrates use of the MOD and INC commands in processor ELD. The LUT is a multi-story frame structure, the floors of which are supported by columns and diagonals. Non-structural mass was attached at the four corners of each floor as rigid lumped mass. Varying cross-sectional properties for the general beam sections were defined throughout the structure. The accompanying figures and table show the section property data set distribution. The geometrical and material properties for the LUT are listed below:

$E = 3.0 \times 10^7$ lb/in^2

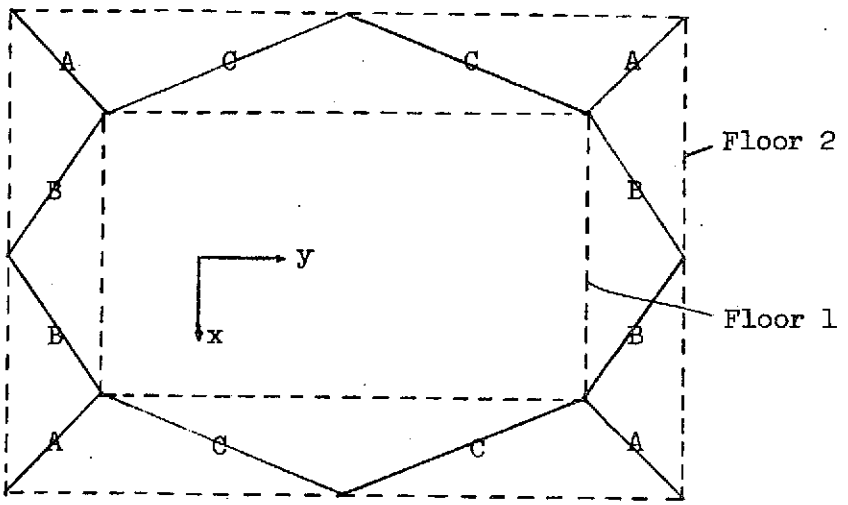
$\nu = 0.2998$

Specific Weight $= 0.28$ lb/in^3

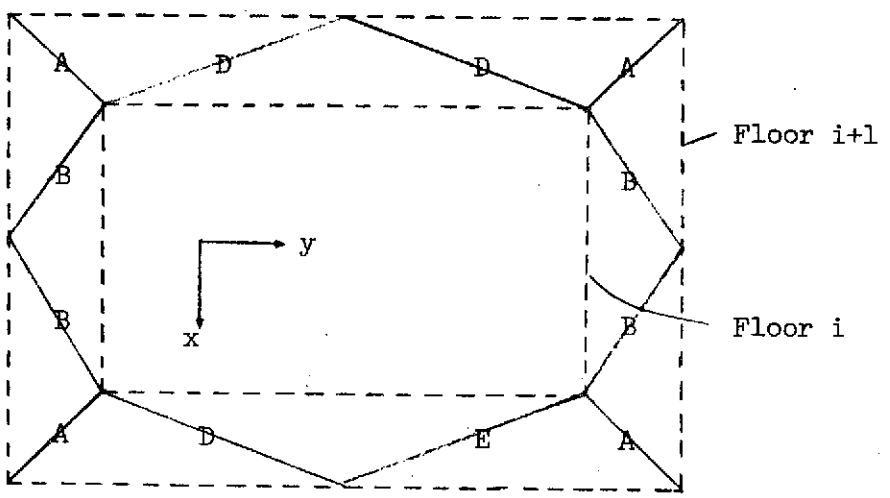


A downward inertia load of 386.088 (equivalent to a one-g loading) was used as a prestress load, and cantilevered vibrational modes were calculated for the prestressed structure.

Floor 1 Bay Layout



Floors 2-3 Bay Layout



Floors 4-19 Layout

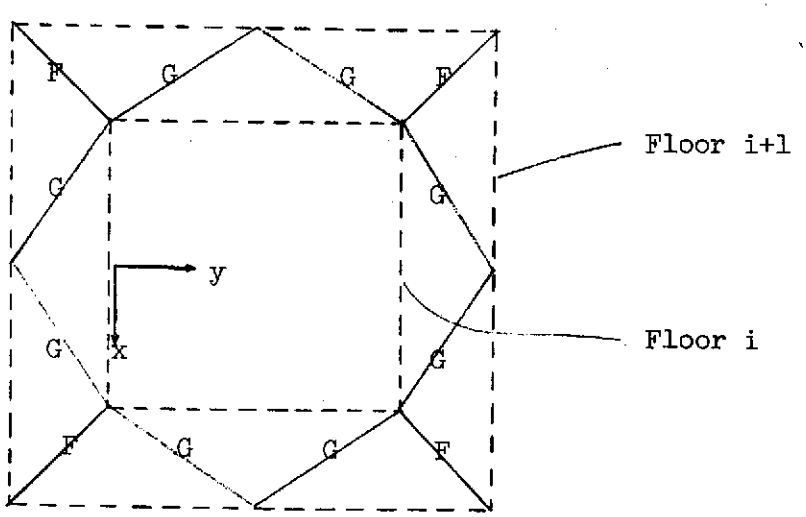
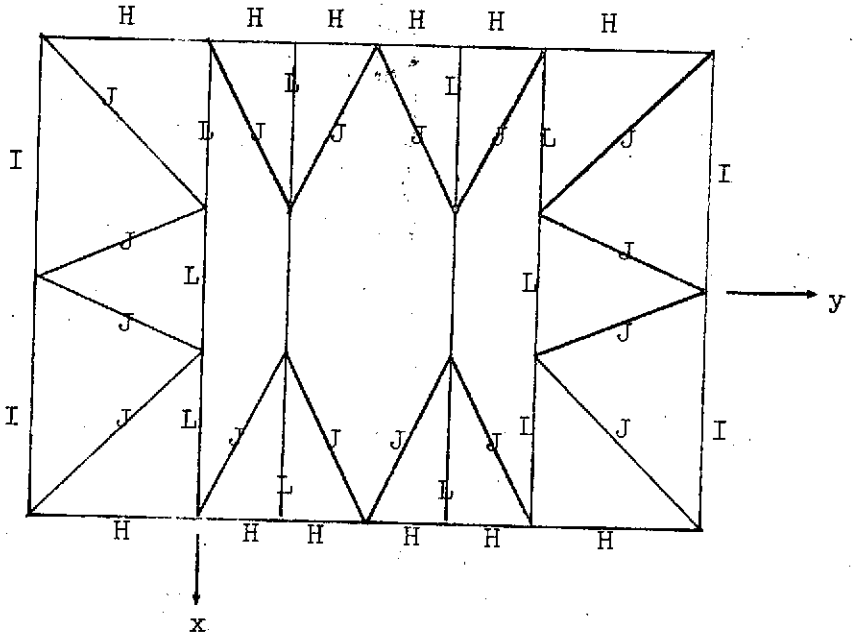


Fig. 10.1

Floors 2-3 Layout



Floors 4-19 Layout

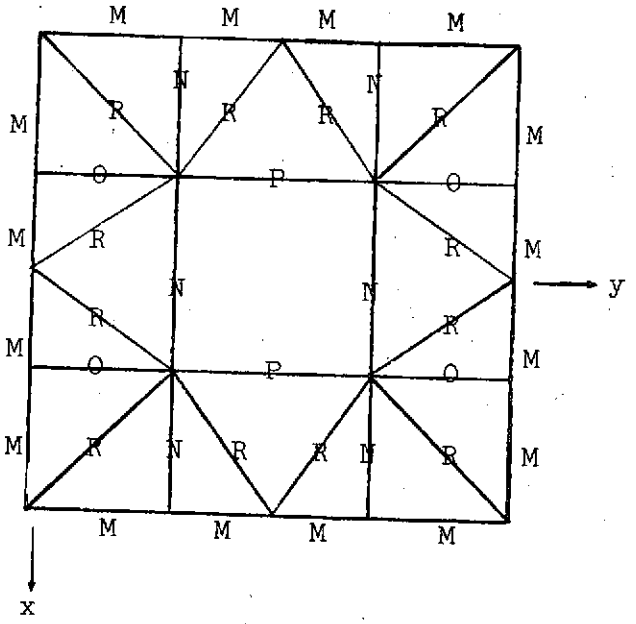


Fig. 10.1 (Continued)

Floor / Bay / Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
A	72	72	76																
B	73	73	77																
C	74																		
D		75	78																
E		72	78																
F				16	19	22	25	28	31	34	37	40	43	46	49	52	55	58	
G				17	20	23	26	29	32	35	38	41	44	47	50	53	56	59	
H		1	6																
I		2	7																
J		3	8																
K		4	9																
L		5	10																
M				18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	66
N				11	11	11	11	11	11	11	11	11	11	11	11	11	11	61	67
O				12	12	12	12	12	12	12	12	12	12	12	12	12	12	62	68
P				13	13	13	13	13	13	13	13	13	13	13	13	13	13	63	69
Q				14	14	14	14	14	14	14	14	14	14	14	14	14	14	64	70
R				15	15	15	15	15	15	15	15	15	15	15	15	15	15	65	71

Table 10.1 Section Data Sets for Floors and Bays

The elastic modes are summarized below:

Mode		ω^2
1	Bending, Direction -2	7.7389
2	Bending, Direction -1	8.9712
3	Torsional	34.3664

```
*XQT TAB                                  . GENERATE BASIC TABLES DEFINING STRUCTURE
START 372$
TITLE!SATURN V LAUNCHER UMBILICAL TOWER (LUT)
TEXT$
!SATURN V LAUNCHER UMBILICAL TOWER (LUT)
!
!DATA DESIGNED TO EXEMPLIFY THE USE OF !INC!
!AND !MOD!. COMMANDS IN THE !ELD! PROCESSOR.
MATERIAL CONSTANTS$
$                                         MATERIAL PROPERTY IS DEFINED WITH A
$                                         WEIGHT DENSITY
      1 3.+7 .29982668 .28$
CONSTRAINT CASE 1$
$                                         JOINTS 1-4 ARE COMPLETELY CONSTRAINED
      ZERO 1 2 3 4 5 6! 1 4 1$
BEAM ORIENTATION SPECIFICATIONS$
      1 1 3 1 0.$
      2 1 1 1 0.$
E21 SECTION PROPERTIES$
      GIVN  1  9012.1  0.   250.4  0.  44.16    9.47$
      GIVN  2  2096.4  0.    76.5  0.  22.4      8.4 $
      GIVN  3    63.35 0.    63.35 0.   7.3    126.7 $
      GIVN  4  4461.   0.   135.1  0.  31.77    4.66$
      GIVN  5  1814.5  0.    63.8  0.  20.      1.7 $
      GIVN  6  9012.1  0.   250.4  0.  44.16    9.47$
      GIVN  7  2096.4  0.    76.5  0.  22.4     8.4 $
      GIVN  8    28.14 0.    28.14 0.   5.58   56.28$
      GIVN  9  2824.8  0.    95.7  0.  24.71    2.57$
      GIVN 10  1140.7  0.    44.   0.  16.18    1.14$
      GIVN 11  1140.7  0.    44.   0.  16.18    1.14$
      GIVN 12    21.7  0.     2.89 0.   3.53     .08$
      GIVN 13   105.3  0.     2.79 0.   4.86     .1 $
      GIVN 14   446.3  0.    22.1  0.  10.59     .5 $
      GIVN 15    15.16 0.    15.16 0.   4.3    30.32$
      GIVN 16 21200.   0. 21200.   0. 276.   31100.  $
      GIVN 17   475.7  0.   475.7  0.  26.04  951.4 $
      GIVN 18  3988.6  0.   116.9  0.  29.11    3.47$
      GIVN 19 16667.   0. 16667.   0. 228.   24554.  $
      GIVN 20   475.7  0.   475.7  0.  26.04  951.4 $
      GIVN 21  3988.6  0.   116.9  0.  29.11    3.47$
      GIVN 22 14284.   0. 14284.   0. 205.   21538.  $
      GIVN 23   475.7  0.   475.7  0.  26.04  951.4 $
      GIVN 24  3266.7  0.   115.1  0.  27.65    3.8 $
      GIVN 25 12609.   0. 12609.   0. 182.   18683.  $
      GIVN 26   361.5  0.   361.5  0.  19.24  723.   $
      GIVN 27  3266.7  0.   115.1  0.  27.65    3.8 $
      GIVN 28 10773.   0. 10773.   0. 160.   16000.  $
      GIVN 29   361.5  0.   361.5  0.  19.24  723.   $
      GIVN 30  3266.7  0.   115.1  0.  27.65    3.8 $
      GIVN 31  9068.   0.  9068.   0. 138.   13476.  $
      GIVN 32   361.5  0.   361.5  0.  19.24  723.   $
      GIVN 33  3266.7  0.   115.1  0.  27.65    3.8 $
      GIVN 34  7459.   0.  7459.   0. 117.   11233.  $
      GIVN 35   361.5  0.   361.5  0.  19.24  723.   $
      GIVN 36  2364.3  0.    88.3  0.  24.7     3.57$
      GIVN 37  5969.   0.  5969.   0.  96.    8917.  $
      GIVN 38   361.5  0.   361.5  0.  19.24  723.   $
      GIVN 39  2364.3  0.    88.3  0.  24.7     3.57$
      GIVN 40  4585.   0.  4585.   0.  76.    6859.  $
      GIVN 41   248.5  0.   248.5  0.  12.88  497.   $
      GIVN 42  2096.4  0.    76.5  0.  22.4     8.4 $
```

GIVN 43	2402.4	0.	930.1	0.	56.73	34.45$	
GIVN 44	248.5	0.	248.5	0.	12.88	497.	$
GIVN 45	2096.4	0.	76.5	0.	22.4	8.4	$
GIVN 46	2402.4	0.	930.1	0.	56.73	34.45$	
GIVN 47	248.5	0.	248.5	0.	12.88	497.	$
GIVN 48	1814.5	0.	63.8	0.	20.	1.7	$
GIVN 49	1266.5	0.	454.9	0.	32.65	7.63$	
GIVN 50	192.3	0.	192.3	0.	9.84	384.6	$
GIVN 51	1814.5	0.	63.8	0.	20.	1.7	$
GIVN 52	1266.5	0.	454.9	0.	32.65	7.63$	
GIVN 53	192.3	0.	192.3	0.	9.84	384.6	$
GIVN 54	1814.5	0.	63.8	0.	20.	1.7	$
GIVN 55	641.5	0.	107.3	0.	17.94	2.01$	
GIVN 56	192.3	0.	192.3	0.	9.84	384.6	$
GIVN 57	1814.5	0.	63.8	0.	20.	1.7	$
GIVN 58	641.5	0.	107.3	0.	17.94	2.01$	
GIVN 59	192.3	0.	192.3	0.	9.84	384.6	$
GIVN 60	1814.5	0.	63.8	0.	20.	1.7	$
GIVN 61	2987.3	0.	203.5	0.	29.43	4.52$	
GIVN 62	1140.7	0.	44.	0.	16.18	1.14$	
GIVN 63	1326.8	0.	53.1	0.	18.23	1.71$	
GIVN 64	446.3	0.	22.1	0.	10.59	.5	$
GIVN 65	15.16	0.	15.16	0.	4.3	30.32$	
GIVN 66	5866.9	0.	170.3	0.	34.71	4.9	$
GIVN 67	2987.3	0.	203.5	0.	29.43	4.52$	
GIVN 68	2987.3	0.	203.5	0.	29.43	4.52$	
GIVN 69	2987.3	0.	203.5	0.	29.43	4.52$	
GIVN 70	2987.3	0.	203.5	0.	29.43	4.52$	
GIVN 71	15.16	0.	15.16	0.	4.3	30.32$	
GIVN 72	26478.	0.	26478.	0.	326.	38584.	$
GIVN 73	562.	0.	562.	0.	18.41	1124.	$
GIVN 74	732.	0.	732.	0.	24.35	1464.	$
GIVN 75	1157.	0.	1157.	0.	40.19	2314.	$
GIVN 76	26300.	0.	26300.	0.	294.	38700.	$
GIVN 77	562.	0.	562.	0.	18.41	1124.	$
GIVN 78	1556.	0.	1556.	0.	56.6	3112.	$

JOINT LOCATIONS$

1	360.	907.	-2880.	270.	589.	-2160.	2 48$
2	-360.	907.	-2880.	-270.	589.	-2160.	2 48$
3	-360.	-427.	-2880.	-270.	-109.	-2160.	2 48$
4	360.	-427.	-2880.	270.	-109.	-2160.	2 48$
5	316.	348.	-2520.	270.	348.	-2160.	2 24$
6	-316.	348.	-2520.	-270.	348.	-2160.	2 24$
7	-316.	132.	-2520.	-270.	132.	-2160.	2 24$
8	316.	132.	-2520.	270.	132.	-2160.	2 24$
9	316.	480.	-2520.	270.	480.	-2160.	2 24$
10	-316.	480.	-2520.	-270.	480.	-2160.	2 24$
11	-316.	0.	-2520.	-270.	0.	-2160.	2 24$
12	316.	0.	-2520.	270.	0.	-2160.	2 24$
13	108.	480.	-2520.	108.	480.	-2160.	2 24$
14	-108.	480.	-2520.	-108.	480.	-2160.	2 24$
15	108.	348.	-2520.	108.	348.	-2160.	2 24$
16	-108.	348.	-2520.	-108.	348.	-2160.	2 24$
17	108.	132.	-2520.	108.	132.	-2160.	2 24$
18	-108.	132.	-2520.	-108.	132.	-2160.	2 24$
19	108.	0.	-2520.	108.	0.	-2160.	2 24$
20	-108.	0.	-2520.	-108.	0.	-2160.	2 24$
21	316.	240.	-2520.	270.	240.	-2160.	2 24$
22	0.	751.	-2520.	0.	569.	-2160.	2 24$
23	-316.	240.	-2520.	-270.	240.	-2160.	2 24$
24	0.	-271.	-2520.	0.	-109.	-2160.	2 24$

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```
        25  316.   751. -2520.$
        26 -316.   751. -2520.$
        27 -316.  -271. -2520.$
        28  316.  -271. -2520.$
        53  240.   348. -1920.   240.   348.  1440. 15 20$
        54  108.   480. -1920.   108.   480.  1440. 15 20$
        55 -108.   480. -1920.  -108.   480.  1440. 15 20$
        56 -240.   348. -1920.  -240.   348.  1440. 15 20$
        57 -240.   132. -1920.  -240.   132.  1440. 15 20$
        58 -108.     0. -1920.  -108.     0.  1440. 15 20$
        59  108.     0. -1920.   108.     0.  1440. 15 20$
        60  240.   132. -1920.   240.   132.  1440. 15 20$
        61  108.   348. -1920.   108.   348.  1440. 15 20$
        62 -108.   132. -1920.  -108.   132.  1440. 15 20$
        63 -108.   348. -1920.  -108.   348.  1440. 15 20$
        64  108.   132. -1920.   108.   132.  1440. 15 20$
        65  240.   240. -1920.   240.   240.  1680. 16 20$
        66    0.   480. -1920.     0.   480.  1680. 16 20$
        67 -240.   240. -1920.  -240.   240.  1680. 16 20$
        68    0.     0. -1920.     0.     0.  1680. 16 20$
        69  240.   480. -1920.   240.   480.  1680. 16 20$
        70 -240.   480. -1920.  -240.   480.  1680. 16 20$
        71 -240.     0. -1920.  -240.     0.  1680. 16 20$
        72  240.     0. -1920.   240.     0.  1680. 16 20$
       353  240.   290.  1680.$
       354   50.   480.  1680.$
       355  -50.   480.  1680.$
       356 -240.   290.  1680.$
       357 -240.   190.  1680.$
       358  -50.     0.  1680.$
       359   50.     0.  1680.$
       360  240.   190.  1680.$
       361   50.   290.  1680.$
       362  -50.   190.  1680.$
       363  -50.   290.  1680.$
       364   50.   190.  1680.$
RMASS$
$                                   RIGID LUMPED MASSES
      REPEAT  4 1$
        49 100.025$
      REPEAT  4 1$
        89  43.880$
      REPEAT  4 1$
       109  24.788$
      REPEAT  4 1$
       129  47.480$
      REPEAT  4 1$
       149  30.632$
      REPEAT  4 1$
       169  64.710$
      REPEAT  4 1$
       189  56.938$
      REPEAT  4 1$
       209  84.460$
      REPEAT  4 1$
       229  72.472$
      REPEAT  4 1$
       249 110.585$
      REPEAT  4 1$
       269 116.048$
      REPEAT  4 1$
```

```
        289 121.750$
        REPEAT  4 1$
        309 106.598$
        REPEAT  4 1$
        329 151.376$
        REPEAT  4 1$
        349 231.478$
        REPEAT  4 1$
        369 308.282$
@XQT ELD                                    . READ ELEMENT DEFINITIONS
E21 $
 GROUP 1        !FLOOR 2
      NSECT=1   :  9 25   1  1 4 1   $
                   5  9   1  1 4 1   $
                   5 21   1  1 2 2   $
                   8 21              $
                   6 23              $
      NSECT=2   : 22 26   1  1 2 2   $
                  22 25   1  1 2 2   $
      NSECT=3   : 13 25   1  1 2 1   $
                  13 22   1  1 2 6   $
                  14 22              $
                  20 27              $
                   9 15   1  1 2 1   $
                  11 18   1  1 2 5   $
                  12 17              $
                  17 21   1  1 2 3   $
                  15 21              $
                  18 23   1  1 2 1   $
      NSECT=4   :  5 15   1  1 2 1   $
                   7 18              $
                   8 17              $
                  15 16   1  1 2 2   $
      NSECT=5   :  9 13   1  1 2 1   $
                  11 20              $
                  12 19              $
                  13 14   1  1 2 6   $
 GROUP 2        !FLOOR 3
   MOD JOINT=24 : MOD NSECT=5 $
      NSECT=1   :  9 25   1  1 4 1   $
                   5  9   1  1 4 1   $
                   5 21   1  1 2 2   $
                   8 21              $
                   6 23              $
      NSECT=2   : 22 26   1  1 2 2   $
                  22 25   1  1 2 2   $
      NSECT=3   : 13 25   1  1 2 1   $
                  13 22   1  1 2 6   $
                  14 22              $
                  20 27              $
                   9 15   1  1 2 1   $
                  11 18   1  1 2 5   $
                  12 17              $
                  17 21   1  1 2 3   $
                  15 21              $
                  18 23   1  1 2 1   $
      NSECT=4   :  5 15   1  1 2 1   $
                   7 18              $
                   8 17              $
                  15 16   1  1 2 2   $
      NSECT=5   :  9 13   1  1 2 1   $
```

```
                11 20                          $
                12 19                          $
                13 14   1  1  2  6            $
  GROUP 3       !FLOORS 4-19
   INC NSECT=3 $
    MOD JOINT=0 ! MOD NSECT=0 $
      NSECT=18 ! 54 66     1  1  15 20 $
      NSECT=18 ! 55 66     1  1  15 20 $
      NSECT=18 ! 55 70     1  1  15 20 $
      NSECT=18 ! 54 69     1  1  15 20 $
      NSECT=18 ! 56 70     1  1  15 20 $
      NSECT=18 ! 56 67     1  1  15 20 $
      NSECT=18 ! 57 67     1  1  15 20 $
      NSECT=18 ! 57 71     1  1  15 20 $
      NSECT=18 ! 58 71     1  1  15 20 $
      NSECT=18 ! 58 68     1  1  15 20 $
      NSECT=18 ! 59 72     1  1  15 20 $
      NSECT=18 ! 59 68     1  1  15 20 $
      NSECT=18 ! 60 72     1  1  15 20 $
      NSECT=18 ! 60 65     1  1  15 20 $
      NSECT=18 ! 53 69     1  1  15 20 $
      NSECT=18 ! 53 65     1  1  15 20 $
    INC NSECT=0 $
     MOD JOINT=300 $
       NSECT=66 ! 54 66     1  1   2  6 $
                  55 66     1  1   2  1 $
                  54 69     1  1   2  1 $
                  56 70     1  1   2  1 $
                  57 67     1  1   2  1 $
                  58 71     1  1   2  1 $
                  59 68                  $
                  60 65                  $
                  53 69                  $
                  53 65                  $
      MOD JOINT=0$
        NSECT=11 ! 53 61     1  1  14 20 $
                   61 63     1  1  14 20 $
                   56 63     1  1  14 20 $
                   57 62     1  1  14 20 $
                   62 64     1  1  14 20 $
                   60 64     1  1  14 20 $
        NSECT=12 ! 55 63     1  1  14 20 $
                   58 62     1  1  14 20 $
                   59 64     1  1  14 20 $
        NSECT=13 ! 62 63     1  1  14 20 $
                   61 64     1  1  14 20 $
        NSECT=14 ! 54 61     1  1  14 20 $
        NSECT=15 ! 61 66     1  1  14 20 $
                   63 66     1  1  14 20 $
                   63 67     1  1  14 20 $
                   62 67     1  1  14 20 $
                   63 70     1  1  14 20 $
                   62 71     1  1  14 20 $
                   62 68     1  1  14 20 $
                   64 68     1  1  14 20 $
                   64 72     1  1  14 20 $
                   64 65     1  1  14 20 $
                   61 65     1  1  14 20 $
                   61 69     1  1  14 20 $
    INC NSECT=6 $
     MOD JOINT=280$
```


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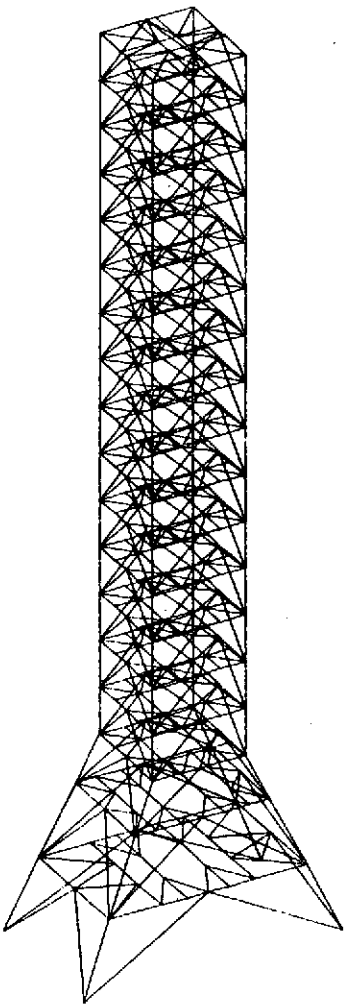
```
      NSECT=61 ! 53 61     1 1   2 20 $
      NSECT=61 ! 61 63     1 1   2 20 $
      NSECT=61 ! 56 63     1 1   2 20 $
      NSECT=61 ! 57 62     1 1   2 20 $
      NSECT=61 ! 62 64     1 1   2 20 $
      NSECT=61 ! 60 64     1 1   2 20 $
      NSECT=62 ! 55 63     1 1   2 20 $
      NSECT=62 ! 58 62     1 1   2 20 $
      NSECT=62 ! 59 64     1 1   2 20 $
      NSECT=63 ! 62 63     1 1   2 20 $
      NSECT=63 ! 61 64     1 1   2 20 $
      NSECT=64 ! 54 61     1 1   2 20 $
      NSECT=65 ! 61 66     1 1   2 20 $
      NSECT=65 ! 63 66     1 1   2 20 $
      NSECT=65 ! 63 67     1 1   2 20 $
      NSECT=65 ! 62 67     1 1   2 20 $
      NSECT=65 ! 63 70     1 1   2 20 $
      NSECT=65 ! 62 71     1 1   2 20 $
      NSECT=65 ! 62 68     1 1   2 20 $
      NSECT=65 ! 64 68     1 1   2 20 $
      NSECT=65 ! 64 72     1 1   2 20 $
      NSECT=65 ! 64 65     1 1   2 20 $
      NSECT=65 ! 61 65     1 1   2 20 $
      NSECT=65 ! 61 69     1 1   2 20 $
  GROUP 4       !INTER-FLOOR COLUMNS AND DIAGONALS
   INC NSECT=0 $
   MOD JOINT=0$
    NREF=2     $
      NSECT=72 !  1 25     1 1   4  1 $
               ! 25 49     1 1   4  1 $
               ! 25 45                $
      NSECT=73 !  2 22     1 1   2 24 $
               !  1 22     1 1   2 24 $
               !  3 24     1 1   2 24 $
               !  4 24     1 1   2 24 $
      NSECT=74 !  2 23                $
               !  4 21                $
               !  1 21     1 1   2  2 $
      NSECT=75 ! 26 47                $
               ! 27 47                $
               ! 28 45                $
      NSECT=76 ! 49 69     1 1   4  1 $
      NSECT=77 ! 49 66     1 1   2  2 $
               ! 50 66     1 1   2  2 $
      NSECT=78 ! 50 67                $
               ! 49 65     1 1   2  2 $
               ! 52 65                $
   INC NSECT=3 $
      NSECT=16 ! 69 89     1 1  15 20   $
      NSECT=16 ! 70 90     1 1  15 20   $
      NSECT=16 ! 71 91     1 1  15 20   $
      NSECT=16 ! 72 92     1 1  15 20   $
      NSECT=17 ! 69 86     1 1  15 20   $
      NSECT=17 ! 70 86     1 1  15 20   $
      NSECT=17 ! 70 87     1 1  15 20   $
      NSECT=17 ! 71 87     1 1  15 20   $
      NSECT=17 ! 71 88     1 1  15 20   $
      NSECT=17 ! 72 88     1 1  15 20   $
      NSECT=17 ! 72 85     1 1  15 20   $
      NSECT=17 ! 69 85     1 1  15 20   $
  @XQT TOPO                            * ANALYZE ELEMENT INTERCONNECTIVITY
```

```
$
@XQT E                                  . FORM ELEMENT DATA PACKETS
$
@XQT EKS                                . INSERT K, S INTO ELEMENT DATA PACKETS
$
@XQT K                                  . FORM SYSTEM K
$
@XQT INV                                . FACTOR K
$
@XQT M                                  . FORM SYSTEM M
$                                         FORM SYSTEM M WITH MASS ENTRIES BY
$                                         CONVERTING MATERIAL PROPERTY WEIGHT
$                                         DENSITY INTO MASS DENSITY
      RESET G=386.088$
@XQT MRM                                . ADD RIGID MASS DATA TO M
$
@XQT Q                                  . DEFINE APPLIED INERTIA PRESTRESS LOADS
      RESET MLIB=1$
      RESET M=M+RM,MTYPE=SPAR$
$                                         INERTIA PRESTRESS LOAD
      CASE 11       INERTIA LOAD FOR PRESTRESS
        INERTIA FORCE 3 -386.088$
@XQT DSOL                               . COMPUTE SOLUTION FOR PRESTRESS LOAD
      RESET MAXIT=2,UP=1,FP=1$
@XQT KG                                 . FORM SYSTEM KG, PRESTRESS SOLUTION
$
@XQT LCM                                . FORM LINEAR COMBINATION OF MATRICES
$
@XQT INV                                . FACTOR K+KG
      RESET K=K+KG$
@XQT EIG                                . SOLVE SYSTEM EIGENPROBLEM
$                                         PRESTRESS VIBRATION ANALYSIS
      RESET INIT=5,NDYN=5,MAXIT=2,REPRT=1$
      RESET K=K+KG,M=M+RM$
@XQT DCU                                . EXECUTE DATA COMPLEX UTILITY PROGRAM
      TOC 1$
```

SATURN V LAUNCHER UMBILICAL TOWER (LUT)

SEQ	RR	DATE	TIME	ER	WORDS	ROWS /BLK	BLK SIZE	TY	DATA SET NAME N1	N2	N3	N4
1	11	061074	112901	0	18	1	18	0	JDF1	BTAB	1	8
2	12	061074	112901	0	372	372	372	0	JREF	BTAB	2	6
3	26	061074	112901	0	12	1	12	1	ALTR	BTAB	2	4
4	27	061074	112901	0	19	1	19	4	NDAL		0	0
5	28	061074	112902	0	80	4	80	3	TEXT	BTAB	2	1
6	31	061074	112902	0	10	1	10	1	MATC	BTAB	2	2
7	32	061074	112903	0	372	372	372	0	CON		1	0
8	46	061074	112903	0	10	2	10	1	MREF	BTAB	2	7
9	47	061074	112906	0	2418	78	2418	1	BA	BTAB	2	9
10	134	061074	112909	0	1116	372	1116	1	JLOC	BTAB	2	5
11	174	061074	112909	0	2232	372	2232	1	RMAS	BTAB	2	18
12	254	061074	112909	0	3348	372	3348	1	QJJT	BTAB	2	19
13	374	061074	112921	0	15104	944	896	0	DEF	E21	1	2
14	918	061074	112921	0	8	4	8	0	GD	E21	1	2
15	919	061074	112921	0	60	4	60	0	GTIT	E21	1	2
16	922	061074	112921	0	12	12	12	0	NELZ	BTAB	1	11
17	923	061074	112921	0	5	1	5	0	KE		0	0
18	924	061074	112921	0	6	1	6	0	NS		0	0
19	925	061074	112921	0	1	1	1	3	ELTS	NAME	0	0
20	926	061074	112921	0	1	1	1	0	ELTS	LTYP	0	-0
21	936	061074	112921	0	1	1	1	0	ELTS	NNOD	0	0
22	937	061074	112921	0	1	1	1	0	ELTS	ISCT	0	0
23	938	061074	112921	0	1	1	1	0	ELTS	NELS	0	0
24	939	061074	112921	0	1	1	1	0	ELTS	LE3	0	0
25	940	061074	112929	0	10752	372	1344	0	KMAP		1316	20
26	1324	061074	112932	0	12096	372	1344	0	AMAP	****	2223	71
27	1756	061074	112957	0	132160	944	140	4	E21		1	2
28	6476	061074	112943	0	20	20	20	0	DIR	E21	1	2
29	6477	061074	113057	0	51520	372	2240	1	K	SPAR	36	1316
30	8317	061074	113129	0	90944	372	3136	1	INV	K	1	2223
31	11565	061074	113242	0	51520	372	2240	1	M	SPAR	36	1316
32	13405	061074	113249	0	51520	372	2240	1	M+RM	SPAR	36	1316
33	15245	061074	113258	0	2232	372	2232	-1	M+RM	DIAG	0	0
34	15325	061074	113303	0	2232	372	2232	-1	RBMX	VEC	1	0
35	15405	061074	113306	0	2232	372	2232	-1	RBMX	VEC	2	0
36	15485	061074	113309	0	2232	372	2232	-1	RBMX	VEC	3	0
37	15565	061074	113310	0	2232	372	2232	-1	NFM		0	1
38	15645	061074	113310	0	15	1	15	4	CASE	****	0	1
39	15646	061074	113310	0	0	0	0	0	LDIR		1	1
40	15646	061074	113331	0	2232	372	2232	1	SSOL	U	1	1
41	15735	061074	213213	0	51520	372	2240	1	KG	SPAR	36	1316
42	17575	061074	213215	0	51520	372	2240	1	K+KG	SPAR	36	1316
43	19415	061074	213303	0	90944	372	3136	1	INV	K+KG	1	2223
44	22663	061074	213311	0	5	5	5	-1	VIBR	EVAL	1	0
45	22664	061074	213312	0	11160	372	2232	-1	VIBR	U	1	0

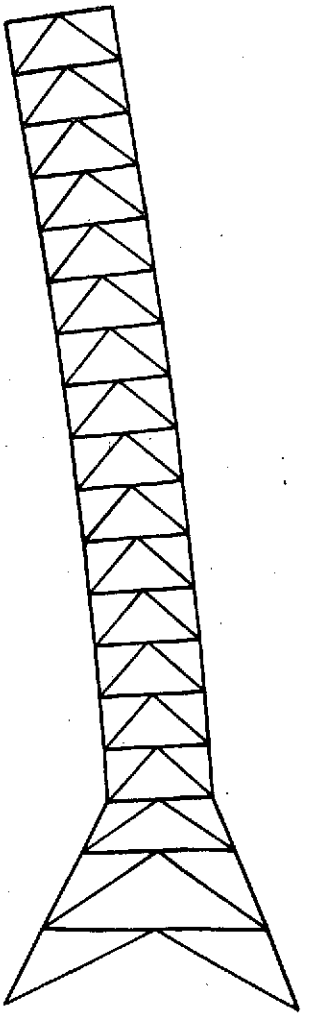
UNDEFORMED STRUCTURE



LAUNCHER UMBILICAL TOWER

SCALE

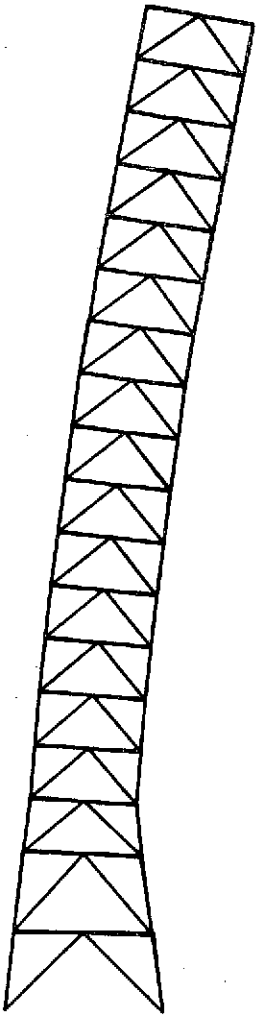
SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = .442753x10^{+00}



LAUNCHER UMBILICAL TOWER

0 84
SCALE

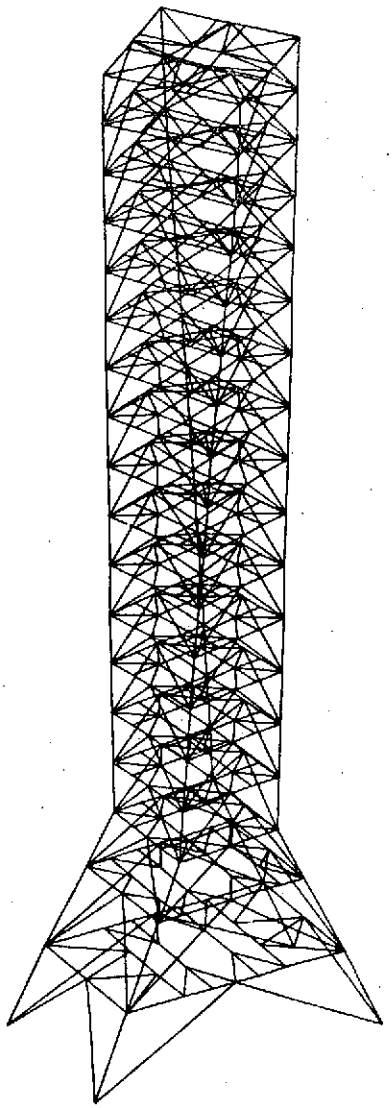
SEQ 2 VIBRATIONAL MODE. FREQ (HZ) = .476699X10^{+00}



LAUNCHER UMBILICAL TOWER

0 844
SCALE

SEQ 3 VIBRATIONAL MODE. FREQ (HZ) = .933012X10^{+00}

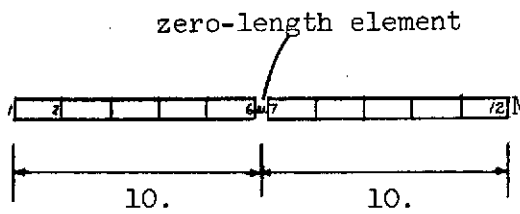


LAUNCHER UMBILICAL TOWER

0 750
SCALE

11. MECHANISM

Free-free vibrational modes were obtained for a linkage in order to demonstrate the application of a spectral shift in the eigensolution process. The linkage was defined as a system of two beam segments connected through a zero-length 6 x 6 directly specified stiffness matrix. Planar motion was specified for the otherwise unconstrained configuration. The geometrical and material properties of the linkage are shown on the figure below:



$E = .1 \times 10^{8}$ in^2

$\nu = .3$

Mass density $= .1 \times 10^{-4}$ $lb\text{-}sec^2/in^4$

$I = 1.0$ in^4

$A = 1.0$ in^2

Eigenvalues were desired in the neighborhood of the value $c = 0.1 \times 10^{11}$, which was selected as the shift point for the eigenvalue solution of K-cM (see Reference 1, Section 3.17). Four "zero" eigenvalues were obtained, representing the three rigid body motions and a mechanism mode.

The elastic modes are summarized below:

	SPAR
ω_1^2	$.24584 \times 10^{11}$
ω_2^2	$.34629 \times 10^{11}$
ω_3^2	$.36038 \times 10^{11}$

```
@XQT TAB                                    . GENERATE BASIC TABLES DEFINING STRUCTURE
START 12   3 4 5$
TITLE!TWO-BEAM LINKAGE
TEXT$
!SYSTEM OF TWO BEAM SEGMENTS CONNECTED
!BY A ZERO-LENGTH ELEMENT.
!
!PROBLEM DESIGNED TO ILLUSTRATE THE USE OF A
!SHIFT POINT IN CALCULATION OF FREE-FREE
!VIBRATIONAL MODES.
MATC$                                       MATERIAL CONSTANTS
     1 1.+7 .3 .1-4$
JLOC$                                       JOINT LOCATIONS
     1  0. 0. 0. 10. 0. 0. 6 1 2$
     6 10. 0. 0. 20. 0. 0. 6 1$
CON 1$                                      CONSTRAINT CASE
MREF$                                       BEAM ORIENTATION SPECIFICATIONS
     1 1 2 1 1.$
E21 SECTION PROPERTIES$
     GIVN 1 1. 1. 1. 1. 1. 1.$
BB$                                         BEAM S6X6 STIFFNESS COEFFICIENTS
$                                           DIRECTLY SPECIFIED STIFFNESS MATRIX,
$                                           THE LAST FOUR ROWS OF WHICH ARE ZERO.
     1 1.+8; 0. 1.+8; ; ; ; $
@XQT ELD                                    . READ ELEMENT DEFINITIONS
E21 $
$                                           BEAM SEGMENT DEFINITIONS
     1 2  1 5 2 6$
E25 $
$                                           ZERO-LENGTH ELEMENT DEFINTION
     6 7$
@XQT TOPO                                   . ANALYZE ELEMENT INTERCONNECTIVITY
@XQT E                                      . FORM ELEMENT DATA PACKETS
@XQT EKS                                    . INSERT K, S INTO ELEMENT DATA PACKETS
@XQT K                                      . FORM SYSTEM K
@XQT M                                      . FORM SYSTEM M
@XQT LCM                                    . FORM LINEAR COMBINATION OF MATRICES
$                                           SHIFT POINT SELECTED = .1+11
$                                           FOR THE EQUATION R=C1*Q1+C2*Q2,
$                                           R=K+LM
$                                           C1=1.0 (DEFAULT)
$                                           Q1=K   (DEFAULT)
$                                           C2=-.1+11
$                                           Q2=M
     RESET R=K+LM,Q2=M,C2=-.1+11$
@XQT INV                                    . FACTOR K+LM
     RESET K=K+LM$
@XQT EIG                                    . SOLVE SYSTEM EIGENPROBLEM
$                                           SOLVE THE EIGENPROBLEM FOR THE !SHIFTED!
$                                           EIGENSYSTEM.
$                                           USE K=K+LM AS THE !SHIFTED MATRIX!.
$                                           ADD  SHIFT=.1+11  TO THE RESULTING
$                                           EIGENVALUES.
     RESET INIT=10,NDYN=10,REPRT=1$
     RESET K=K+LM$
     RESET SHIFT=.1+11$
     PRINT 0 0 0 1 1$
@XQT DCU                                    . EXECUTE DATA COMPLEX UTILITY PROGRAM
     TOC 1$
```

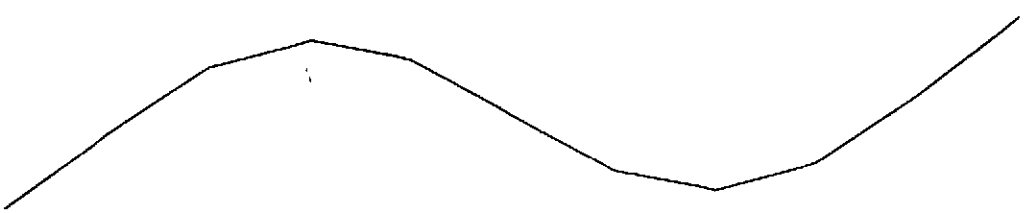
TABLE OF CONTENTS, LIBRARY 1

TWO-BEAM LINKAGE

SEQ	RR	DATE	TIME	E R	WORDS	ROWS /BLK	BLK SIZE	T Y	DATA SET NAME N1	N2	N3	N4
1	11	062074	001823	0	18	1	18	0	JDF1	BTAB	1	8
2	12	062074	001823	0	12	12	12	0	JREF	BTAB	2	6
3	13	062074	001823	0	12	1	12	1	ALTR	BTAB	2	4
4	14	062074	001823	0	19	1	19	4	NDAL		0	0
5	15	062074	001823	0	120	6	120	3	TEXT	BTAB	2	1
6	20	062074	001824	0	10	1	10	1	MATC	BTAB	2	2
7	21	062074	001824	0	36	12	36	1	JLOC	BTAB	2	5
8	23	062074	001824	0	12	12	12	0	CON		1	0
9	24	062074	001824	0	5	1	5	1	MREF	BTAB	2	7
10	25	062074	001824	0	31	1	31	1	BA	BTAB	2	9
11	27	062074	001824	0	21	1	21	1	BB	BTAB	2	10
12	28	062074	001824	0	108	12	108	1	GJJT	BTAB	2	19
13	32	062074	001826	0	160	10	896	0	DEF	E21	1	2
14	64	062074	001826	0	2	1	2	0	GD	E21	1	2
15	65	062074	001826	0	15	1	15	0	GTIT	E21	1	2
16	66	062074	001826	0	16	1	896	0	DEF	E25	5	2
17	98	062074	001826	0	2	1	2	0	GD	E25	5	2
18	99	062074	001826	0	15	1	15	0	GTIT	E25	5	2
19	100	062074	001826	0	12	12	12	0	NELZ	BTAB	1	11
20	101	062074	001826	0	10	2	10	0	KE		0	0
21	111	062074	001826	0	12	2	12	0	NB		0	0
22	112	062074	001826	0	2	2	2	3	ELTS	NAME	0	0
23	113	062074	001826	0	2	2	2	0	ELTS	LTYP	0	0
24	114	062074	001826	0	2	2	2	0	ELTS	NNOD	0	0
25	115	062074	001826	0	2	2	2	0	ELTS	ISCT	0	0
26	116	062074	001826	0	2	2	2	0	ELTS	NELS	0	0
27	117	062074	001827	0	2	2	2	0	ELTS	LES	0	0
28	118	062074	001828	0	1344	12	1344	0	KMAP		25	3
29	166	062074	001828	0	1344	12	1344	0	AMAP	0000	25	3
30	214	062074	001831	0	1400	10	140	4	E21		1	2
31	264	062074	001830	0	20	20	20	0	DIR	E21	1	2
32	265	062074	001831	0	112	1	112	4	E25		5	2
33	269	062074	001830	0	20	20	20	0	DIR	E25	5	2
34	270	062074	001833	0	2240	12	2240	1	K	SPAR	9	23
35	350	062074	001834	0	2240	12	2240	1	M	SPAR	9	23
36	430	062074	001835	0	2240	12	2240	1	K*LM	SPAR	9	23
37	510	062074	001837	0	3136	12	3136	1	INV	K*LM	1	23
38	622	062074	001839	0	10	10	10	-1	VIBR	EVAL	1	0
39	623	062074	001839	0	720	12	72	-1	VIBR	U	1	0

REPRODUCIBILITY OF THE ORIGINAL PAGE IS POOR

SEQ 5 VIBRATIONAL MODE. EIGENVALUE = $.458397 \times 10^{+10}$

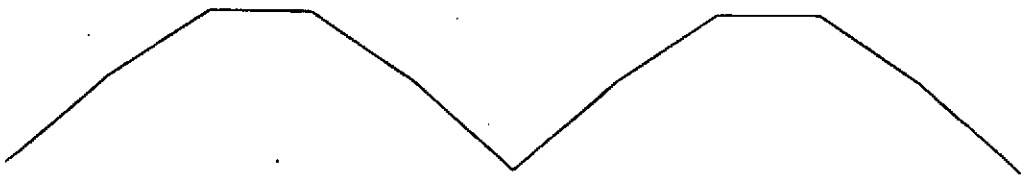


BEAM MECHANISM

SEQ 6 VIBRATIONAL MODE. EIGENVALUE = $.146287 \times 10^{+11}$

BEAM MECHANISM

SEQ 7 VIBRATIONAL MODE. EIGENVALUE = $.160383 \times 10^{+11}$



BEAM MECHANISM

REFERENCES

1. Whetstone, W. D., "SPAR Reference Manual", LMSC-D403168, June 1974

2. Timoshenko, S. and D. H. Young, "Vibration Problems in Engineering", Third Edition, Van Nostrand, 1955

3. Timoshenko, S. and J. M. Gere, "Theory of Elastic Stability", 2nd Edition, McGraw-Hill, 1961

4. Timoshenko, S. and S. Woinowsky-Krieger, "Theory of Plates and Shells", 2nd Edition, McGraw-Hill, 1959

5. Flügge, W. "Stresses in Shells", 2nd printing, Springer-Verlag, 1962

6. Forsberg, K. "Influence of Boundary Conditions on the Modal Characteristics of Thin Cylindrical Shells", AIAA Journal, Vol. 2, No. 12, December 1964

7. Kalnins, A., "Free Vibration of Rotationally Symmetric Shells", The Journal of the Acoustic Society of America, Vol. 36, No. 7, July 1964

8. Huang, N., "Unsymmetrical Buckling of Thin Shallow Spherical Shells", Journal of Applied Mechanics, September 1964

9. Timoshenko, S., "Strength of Materials", Part 2, Third Edition, Van Nostrand, 1956

10. Timoshenko, S. and J. N. Goodier, "Theory of Eleasticity", McGraw-Hill, New York, 1951

11. Seide, "On the Free Vibration of Simply Supported Truncated Conical Shells", SSD-TDR-64-15, 27 February 1964

CORRECTIONS FOR REPORT

LMSC-D403169

CORRECTIONS FOR REPORT

LMSC-D403169

```
$                                          CIRCULAR MEMBRANE UNDER CONSTRAINT CASE 1
    RESET K=K+KG,INIT=8$
PRINT 0 0 0 0 1$
OXQT INV
    RESET K=K+KG,CON=2$
OXQT EIG
$                                          FIND VIBRATIONAL MODES OF THE STRESSED
$                                          CIRCULAR MEMBRANE UNDER CONSTRAINT CASE 2
    RESET K=K+KG,INIT=8,CON=2$
    PRINT 0 0 0 0 -1$
OXQT OCU                                   , EXECUTE DATA COMPLEX UTILITY PROGRAM
    TOC 1$
```

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REPRODUCIBILITY OF THE
ORIGINAL PAGE IS POOR

TABLE OF CONTENTS, DAL 1

VIBRATION OF A CIRCULAR MEMBRANE

SEQ	RR	DATE	TIME	ER	WORDS	ROWS /BLK	BLK SIZE	TY	DATA SET NAME N1	N2	N3	N4
1	11	051474	042706	0	18	1	18	0	JDF1	BTAB	1	8
2	-12	051474	042706	0	101	101	101	0	JREF	BTAB	2	6
3	16	051474	042706	0	12	1	12	1	ALTR	BTAB	2	4
4	17	051474	042706	0	19	1	19	4	NDAL		0	0
5	18	051474	042706	0	10	1	10	1	MATC	BTAB	2	2
6	19	051474	042706	0	101	101	101	0	JREF	BTAB	2	6
7	23	051474	042707	0	303	101	303	1	JLOC	BTAB	2	5
8	34	051474	042707	0	19	1	19	1	SA	BTAB	2	13
9	35	051474	042707	0	101	101	101	0	CON.		1	0
10	39	051474	042708	0	101	101	101	0	CON		2	0
11	43	051474	042709	0	101	101	101	0	CON		3	0
12	47	051474	042709	0	909	101	909	1	QJJT	BTAB	2	19
13	80	051474	042715	0	1296	81	896	0	DEF	E41	9	4
14	144	051474	042715	0	2	1	2	0	GD	E41	9	4
15	145	051474	042715	0	15	1	15	0	GTIT	E41	9	4
16	146	051474	042715	0	144	9	896	0	DEF	E31	6	3
17	178	051474	042715	0	2	1	2	0	GD	E31	6	3
18	179	051474	042715	0	15	1	15	0	GTIT	E31	6	3
19	180	051474	042715	0	12	12	12	0	NELZ	BTAB	1	11
20	181	051474	042715	0	10	2	10	0	KE		0	0
21	191	051474	042715	0	12	2	12	0	NS		0	0
22	192	051474	042716	0	2	2	2	3	ELTS	NAME	0	0
23	193	051474	042716	0	2	2	2	0	ELTS	LTYP	0	0
24	194	051474	042717	0	2	2	2	0	ELTS	NNOD	0	0
25	195	051474	042717	0	2	2	2	0	ELTS	ISCT	0	0
26	196	051474	042718	0	2	2	2	0	ELTS	NELS	0	0
27	197	051474	042718	0	2	2	2	0	ELTS	LE3	0	0
28	198	051474	042721	0	2688	101	1344	0	KMAP		453	30
29	294	051474	042726	0	9408	101	1344	0	AMAP	@@@1	1137	78
30	630	051474	042734	0	1260	9	140	4	E31		6	3
31	675	051474	042730	0	20	20	20	0	DIR	E31	6	3
32	676	051474	042736	0	13608	81	168	4	E41		9	4
33	1162	051474	042732	0	20	20	20	0	DIR	E41	9	4
34	1163	051474	042743	0	6720	101	2240	1	M	SPAR	9	453
35	1403	051474	042745	0	606	101	606	-1	NFM		0	1
36	1425	051474	042745	0	15	1	15	4	CASE	@@@@	0	1
37	1426	051474	042745	0	0	0	0	0	LDIR		1	1
38	1426	051474	042751	0	6720	101	2240	1	K	SPAR	9	453
39	1666	051474	042755	0	9408	101	3136	1	INV	K	3	1137
40	2002	051474	042758	0	606	101	606	1	SSOL	U	0	1
41	2033	051474	042806	0	6720	101	2240	1	KG	SPAR	9	453
42	2273	051474	042807	0	6720	101	2240	1	K*KG	SPAR	9	453
43	2513	051474	042815	0	12544	101	3136	1	INV	K+KG	1	1137
44	2961	051474	042816	0	8	8	8	-1	VIBR	EVAL	1	0
45	2962	051474	042817	0	4848	101	606	-1	VIBR	U	1	0
46	3138	051474	042919	0	12544	101	3136	1	INV	K+KG	2	1137
47	3586	051474	042921	0	8	8	8	-1	VIBR	EVAL	2	0
48	3587	051474	042922	0	4848	101	606	-1	VIBR	U	2	0

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```
@XQT TAB                                . GENERATE BASIC TABLES DEFINING STRUCTURE
START 11$
TITLE' BEAM PROBLEMS
TEXT$
!      THE FOLLOWING BEAM PROBLEMS ARE SOLVED IN THIS RUN:
!       1  VIBRATION OF A CANTILEVERED BEAM
!       2  BUCKLING OF A COLUMN
!       3  LATERAL DEFLECTION OF A COMPRESSED BEAM
!       4  VIBRATION OF A PRESTRESSED BEAM
!       5  LATERAL BUCKLING OF A CANTILEVERED BEAM
!       6  LATERAL BUCKLING OF A BEAM IN PURE BENDING
!       7  BUCKLING OF A BEAM DUE TO TEMPERATURE
!       8  VIBRATION OF A HEATED BEAM
!
MATERIAL CONSTANTS
    1  .1+8  .0  .1  .1=4$
JOINT LOCATIONS
    1  .0  .0  .0  .0  .0  10.0  11 1$
BEAM ORIENTATION SPECIFICATIONS$
    1  1  2  1  0.$
E21 SECTION PROPERTIES$
    GIVN  1  .0004  .0  .0036  .0  .12  .0016$
CONSTRAINT CASE 1$
$                                       THE BEAM IS CONSTRAINED AT ONE END
    FIXED PLANE = 3$
CONSTRAINT CASE 2$
$                                       SIMPLY SUPPORTED AT BOTH ENDS
    ZERO 1,2,3,6:   1,11,10$
@XQT ELD                                . READ ELEMENT DEFINITIONS
E21 $
    1 2  1 10$
@XQT TOPO                               . ANALYZE ELEMENT INTERCONNECTIVITY
$
@XQT E                                  . FORM ELEMENT DATA PACKETS
$
@XQT EKS                                . INSERT K. S INTO ELEMENT DATA PACKETS
$
@XQT M                                  . FORM SYSTEM M
$                                       EXCLUDE ROTATORY INERTIA EFFECTS
    RESET IBEAM=1$
@XQT K                                  . FORM SYSTEM K
$
@XQT INV                                . FACTOR MATRIX IN K SPAR FORMAT
$
@XQT EIG                                . SOLVE SYSTEM EIGENPROBLEM
$                                       FIND VIBRATIONAL MODES OF THE CANTILEVER
$                                       BEAM.
    RESET INIT=10,REPR=1$
@XQT Q                                  . DEFINE APPLIED LOADS
    CASE 1' AXIAL COMPRESSIVE FORCE APPLY AT FREE END.
      NODAL FORCES, MOMENTS
        11 3 -1.0$
    CASE 2' SHEAR FORCE IN 1-DIRECTION APPLY AT FREE END.
      NODAL FORCES, MOMENTS
        11 1 1.0$
    CASE 3' BENDING MOMENT IN 2-DIRECTION APPLY AT BOTH ENDS.
      NODAL FORCES, MOMENTS
        1 5 1.0:  11 5 -1.0$
    CASE 4' CONCENTRATED FORCE AT MID-SPAN
      NODAL FORCES, MOMENTS
        6 1 10.0$
```

```
      CASE 51 TEMPERATURE LOAD AT ALL JOINTS.
        NODAL TEMPERATURES
            1   1.0   11  1$
 @XQT DSOL                              . SOLVE LOAD CASE 1
      RESET L1=1,L2=1$
 @XQT KG                                . FORM SYSTEM KG, AXIAL COMPRESSION
 $
 @XQT EIG
 $                                        FIND BUCKLING LOAD FOR CANTILEVERED
 $                                        COLUMN
      RESET INIT=10,PROB=STAB$
 @XQT LCM                               . FORM LINEAR COMBINATION OF MATRICES
 $                                        FORM STIFFNESS MATRIX FOR AXIALY PRE-
 $                                        COMPRESSED BEAM, COMPRESSIVE FORCE=1000.
      RESET R=KXKG,Q1=K,Q2=KG,C2=1000.0$
 @XQT INV
      RESET K=KXKG,CON=2$
 @XQT DSOL                              . SOLVE LOAD CASE 4
 $                                        FIND DEFLECTION OF THE PRESTRESSED BEAM
 $                                        UNDER LOAD CASE 4.
      RESET K=KXKG,CON=2,L1=4,L2=4$
 @XQT LCM
 $                                        FORM STIFFNESS MATRIX FOR AXIALY PRE-
 $                                        STRESSED BEAM, TENSILE FORCE=1000.
      RESET R=KYKG,Q1=K,Q2=KG,C2=-1000.0$
 @XQT INV
      RESET K=KYKG,CON=2$
 @XQT EIG
 $                                        FIND FREQUENCY FOR FREE VIBRATION OF THE
 $                                        PRE-STRESSED BEAM.
      RESET K=KYKG,CON=2,INIT=10$
 @XQT DSOL                              . SOLVE LOAD CASE 2
      RESET L1=2,L2=2$
 @XQT KG                                . FORM SYSTEM KG, LOAD CASE 2
      RESET QSEQ=2$
 @XQT EIG
 $                                        FIND LATERAL BUCKLING VALUE OF LOAD CASE
 $                                        2 FOR THE CANTILEVER BEAM.
      RESET INIT=6,PROB=STAB$
 @XQT INV
      RESET CON=2$
 @XQT DSOL
      RESET CON=2,L1=3,L2=3$
 @XQT KG                                . FORM SYSTEM KG, PURE BENDING
      RESET QSEQ=3$
 @XQT EIG
 $                                        FIND BUCKLING LOAD OF A SIMPLY SUPPORTED
 $                                        BEAM IN PURE BENDING.
      RESET INIT=6,PROB=STAB,CON=2$
 @XQT DSOL
      RESET L1=5,L2=5,CON=2$
 @XQT KG                                . FORM SYSTEM KG, TEMPERATURE LOAD
      RESET QSEQ=5$
 @XQT EIG
 $                                        FIND BUCKLING TEMPERATURE LOAD FOR A
 $                                        SIMPLY SUPPORTED BEAM.
      RESET CON=2,INIT=6,PROB=STAB$
 @XQT LCM
      RESET R=K5KG,Q1=K,Q2=KG,C2=10.0$
 @XQT INV
      RESET K=K5KG,CON=2$
```

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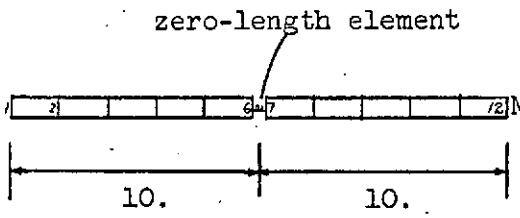
11. MECHANISM

Free-free vibrational modes were obtained for a linkage in order to demonstrate the application of a spectral shift in the eigensolution process. The linkage was defined as a system of two beam segments connected through a zero-length 6 x 6 directly specified stiffness matrix. Planar motion was specified for the otherwise unconstrained configuration. The geometrical and material properties of the linkage are shown on the figure below:



$E = .1 \times 10^{8}$ in^2

$\nu = .3$

Mass density $= .1 \times 10^{-4}$ lb-sec^2/in^4

$I = 1.0$ in^4

$A = 1.0$ in^2

Eigenvalues were desired in the neighborhood of the value $c = 0.1 \times 10^{11}$, which was selected as the shift point for the eigenvalue solution of K-cM (see Reference 1, Section 3.17). Four "zero" eigenvalues were obtained, representing the three rigid body motions and a mechanism mode.

The elastic modes are summarized below:

	SPAR
ω_1^2	$.24584 \times 10^{11}$
ω_2^2	$.34629 \times 10^{11}$
ω_3^2	$.36038 \times 10^{11}$

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```
@XQT TAB                                    . GENERATE BASIC TABLES DEFINING STRUCTURE
START 12   3 4 5$
TITLE'TWO-BEAM LINKAGE
TEXT$
'SYSTEM OF TWO BEAM SEGMENTS CONNECTED
'BY A ZERO-LENGTH ELEMENT.
'
'PROBLEM DESIGNED TO ILLUSTRATE THE USE OF A
'SHIFT POINT IN CALCULATION OF FREE-FREE
'VIBRATIONAL MODES.
MATC$                                       MATERIAL CONSTANTS
     1 1.+7 .3 .1-4$
JLOC$                                       JOINT LOCATIONS
     1  0. 0. 0. 10. 0. 0. 6 1 2$
     6 10. 0. 0. 20. 0. 0. 6 1$
CON 1$                                      CONSTRAINT CASE
MREF$                                       BEAM ORIENTATION SPECIFICATIONS
     1 1 2 1 1.$
E21 SECTION PROPERTIES$
     GIVN 1 1. 1. 1. 1. 1. 1.$
BB$                                         BEAM S6X6 STIFFNESS COEFFICIENTS
$                                           DIRECTLY SPECIFIED STIFFNESS MATRIX,
$                                           THE LAST FOUR ROWS OF WHICH ARE ZERO.
     1 1.+8; 0. 1.+8; ; ; 1 $
@XQT ELD                                    . READ ELEMENT DEFINITIONS
E21 $
$                                           BEAM SEGMENT DEFINITIONS
     1 2  1 5 2 6$
E25 $
$                                           ZERO-LENGTH ELEMENT DEFINTION
     6 7$
@XQT TOPO                                   . ANALYZE ELEMENT INTERCONNECTIVITY
@XQT E                                      . FORM ELEMENT DATA PACKETS
@XQT EKS                                    . INSERT K, S INTO ELEMENT DATA PACKETS
@XQT K                                      . FORM SYSTEM K
@XQT M                                      . FORM SYSTEM M
@XQT LCM                                    . FORM LINEAR COMBINATION OF MATRICES
$                                           SHIFT POINT SELECTED = .1+11
$                                           FOR THE EQUATION R=C1*Q1+C2*Q2,
$                                           R=K+LM
$                                           C1=1.0  (DEFAULT)
$                                           Q1=K    (DEFAULT)
$                                           C2=-.1+11
$                                           Q2=M
     RESET R=K+LM,Q2=M,C2=-.1+11$
@XQT INV                                    . FACTOR K+LM
     RESET K=K+LM$
@XQT EIG                                    . SOLVE SYSTEM EIGENPROBLEM
$                                           SOLVE THE EIGENPROBLEM FOR THE 'SHIFTED'
$                                           EIGENSYSTEM.
$                                           USE K=K+LM AS THE 'SHIFTED MATRIX'.
$                                           ADD  SHIFT=.1+11  TO THE RESULTING
$                                           EIGENVALUES.
     RESET INIT=10,NDYN=10,REPRT=1$
     RESET K=K+LM$
     RESET SHIFT=.1+11$
     PRINT 0 0 0 1 1$
@XQT DCU                                    . EXECUTE DATA COMPLEX UTILITY PROGRAM
     TOC 1$
```

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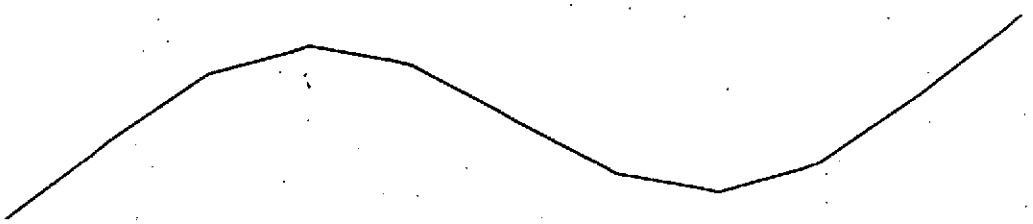
TWO-BEAM LINKAGE

SEQ	RR	DATE	TIME	ER	WORDS	ROWS /BLK	BLK SIZE	TY	DATA SET NAME N1	N2	N3	N4
1	11	062074	001823	0	18	1	18	0	JOF1	BTAB	1	8
2	12	062074	001823	0	12	12	12	0	JREF	BTAB	2	6
3	13	062074	001823	0	12	1	12	1	ALTR	BTAB	2	4
4	14	062074	001823	0	19	1	19	4	NDAL		0	0
5	15	062074	001823	0	120	6	120	3	TEXT	BTAB	2	1
6	20	062074	001824	0	10	1	10	1	MATC	BTAB	2	2
7	21	062074	001824	0	36	12	36	1	JLOC	BTAB	2	5
8	23	062074	001824	0	12	12	12	0	CON		1	0
9	24	062074	001824	0	5	1	5	1	MREF	BTAB	2	7
10	25	062074	001824	0	31	1	31	1	BA	BTAB	2	9
11	27	062074	001824	0	21	1	21	1	BB	BTAB	2	10
12	28	062074	001824	0	108	12	108	1	QJJT	BTAB	2	19
13	32	062074	001826	0	160	10	896	0	DEF	E21	1	2
14	64	062074	001826	0	2	1	2	0	GD	E21	1	2
15	65	062074	001826	0	15	1	15	0	GTIT	E21	1	2
16	66	062074	001826	0	16	1	896	0	DEF	E25	5	2
17	98	062074	001826	0	2	1	2	0	GD	E25	5	2
18	99	062074	001826	0	15	1	15	0	GTIT	E25	5	2
19	100	062074	001826	0	12	12	12	0	NELZ	BTAB	1	11
20	101	062074	001826	0	10	2	10	0	KE		0	0
21	111	062074	001826	0	12	2	12	0	NS		0	0
22	112	062074	001826	0	2	2	2	3	ELTS	NAME	0	0
23	113	062074	001826	0	2	2	2	0	ELTS	LTYP	0	0
24	114	062074	001826	0	2	2	2	0	ELTS	NNOD	0	0
25	115	062074	001826	0	2	2	2	0	ELTS	ISET	0	0
26	116	062074	001826	0	2	2	2	0	ELTS	NELS	0	0
27	117	062074	001827	0	2	2	2	0	ELTS	LEN	0	0
28	118	062074	001828	0	1344	12	1344	0	KMAP		21	3
29	166	062074	001828	0	1344	12	1344	0	AMAP	0000	21	3
30	214	062074	001831	0	1400	10	140	4	E21		1	2
31	264	062074	001830	0	20	20	20	0	DIR	E21	1	2
32	265	062074	001831	0	112	1	112	4	E25		5	2
33	269	062074	001830	0	20	20	20	0	DIR	E25	5	2
34	270	062074	001833	0	2240	12	2240	1	K	SPAR	0	23
35	350	062074	001834	0	2240	12	2240	1	M	SPAR	0	23
36	430	062074	001835	0	2240	12	2240	1	K*LM	SPAR	0	23
37	510	062074	001837	0	3136	12	3136	1	INV	K*LM	1	23
38	622	062074	001839	0	10	10	10	-1	VIBR	EVAL	1	0
39	623	062074	001839	0	720	12	72	-1	VIBR	U	1	0

SEQ 5 VIBRATIONAL MODE. EIGENVALUE = $.458397 \times 10^{+10}$



BEAM MECHANISM

0 SCALE